Review Limits name Key

1. $\lim_{n\to\infty} \frac{3n^3 - 5n}{n^3 - 2n^2 + 1}$ is

(A) -5 (B) -2 (C) 1

(D) 3 (E) nonexistent

2. If the function f is continuous for all real numbers and if $f(x) = \frac{x^2 - 4}{x+2}$ when $x \neq -2$, then $f(-2) =$

$\frac{(x-2)\cancel{(x+2)}}{\cancel{x+2}}$

(A) -4 (B) -2 (C) -1

(D) 0 (E) 2

3. $\lim_{\theta\to 0} \frac{1-\cos\theta}{2\sin^2\theta}$ is $\frac{1-\cos\theta}{2(1-\cos^2\theta)} = \frac{\cancel{1-\cos\theta}}{2\cancel{(1-\cos\theta)}(1+\cos\theta)} = \frac{1}{2(1+\cos\theta)}$

(A) 0 (B) $\frac{1}{8}$ (C) $\frac{1}{4}$ $\frac{1}{2(1+1)}$

(D) 1 (E) nonexistent

4. If f is a differentiable function, then $f'(a)$ is given by which of the following?

I. $\lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$

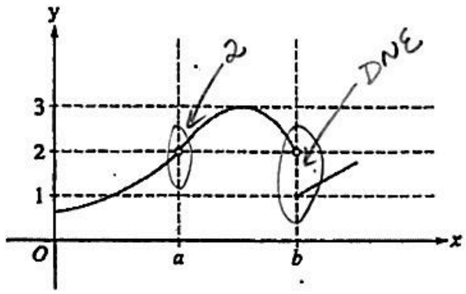
II. $\lim_{x\to a} \frac{f(x) - f(a)}{x-a}$

~~III.~~ $\lim_{x\to a} \frac{f(x+h) - f(x)}{h}$

(A) I only (B) II only (C) I and II only

(D) I and III only (E) I, II, and III

5.



The graph of the function f is shown in the figure above. Which of the following statements about f is true?

(A) $\lim_{x \to a} f(x) = \lim_{x \to b} f(x)$

(B) $\lim_{x \to a} f(x) = 2$

(C) $\lim_{x \to b} f(x) = 2$

(D) $\lim_{x \to b} f(x) = 1$

(E) $\lim_{x \to a} f(x)$ does not exist.

6. Let f be the function defined by the following. For what values of x is f NOT continuous?

$$f(x) = \begin{cases} \sin x, & x < 0 \\ x^2, & 0 \le x < 1 \\ 2 - x, & 1 \le x < 2 \\ x - 3, & x \ge 2 \end{cases}$$

$\sin 0 = 0$ ✓
0 1 ✓
1 0 ✗
-1

(A) 0 only
(B) 1 only
(C) 2 only
(D) 0 and 2 only
(E) 0, 1, and 2

7. If $f(x) = \begin{cases} \ln x & \text{for } 0 < x \le 2 \\ x^2 \ln 2 & \text{for } 2 < x \le 4, \end{cases}$ then $\lim_{x \to 2} f(x)$ is

$\ln 2 \ne 4 \ln 2$

(A) ln 2

(B) ln 8

(C) ln 16

(D) 4

(E) nonexistent

8. If $f(x) = \sin(e^{-x})$, then $f'(x) =$

chain rule

sin ( )	cos (
$e^{( )}$	$e^{( )}$
$-x$	-1

$-1\ e^{-x} \cos e^{-x}$

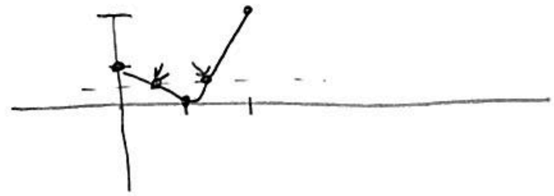
(A) $-\cos(e^{-x})$

(B) $\cos(e^{-x}) + e^{-x}$

(C) $\cos(e^{-x}) - e^{-x}$

(D) $e^{-x} \cos(e^{-x})$

(E) $-e^{-x} \cos(e^{-x})$



x	0	1	2
$f(x)$	1	k	2

9.The function f is continuous on the closed interval [0, 2] and has values that are given in the table above. The equation $f(x) = \frac{1}{2}$ must have at least two solutions in the interval [0, 2] if $k =$

(A) 0 (B) $\frac{1}{2}$ (C) 1

(D) 2 (E) 3

*10. If $a \neq 0$, then $\lim_{x \to a} \frac{x^2 - a^2}{x^4 - a^4}$ is

$$\frac{\cancel{x^2-a^2}}{(\cancel{x^2-a^2})(x^2+a^2)} = \frac{1}{x^2+a^2}$$

(A) $\frac{1}{a^2}$ (B) $\frac{1}{2a^2}$ (C) $\frac{1}{6a^2}$

$$\frac{1}{a^2+a^2} = \frac{1}{2a^2}$$

(D) 0 (E) nonexistent

11. Find g(x),that will make the function continuous at x =1.

$$f(x) = \begin{cases} 2x^2+3 & if\ x \geq 1 \\ g(x) & if\ x < 1 \end{cases}$$

$2(1)^2 + 3 = g(1)$

$g(1) = 5$

a. $\cos(x+4)$ ×

b. x ×

c. 6-x $6-1=5$ ✓

d. $2x^2 - 3$

e. $x^2 + 2$

a) $\lim_{x \to 0^-} f(x) = 1$ $\lim_{x \to 0^+} f(x) = 1$

$\therefore \lim_{x \to 0} f(x) = 1$

$f(0) = 1 - 2\sin(0) = 1$

$f(0) = \lim_{x \to 0} f(x)$

$\therefore f(x)$ is continuous @ $x = 0$

Question 6

Let f be a function defined by $f(x) = \begin{cases} 1 - 2\sin x & \text{for } x \leq 0 \\ e^{-4x} & \text{for } x > 0. \end{cases}$

(a) Show that f is continuous at $x = 0$.

(b) For $x \neq 0$, express $f'(x)$ as a piecewise-defined function. Find the value of x for which $f'(x) = -3$.

(c) Find the average value of f on the interval $[-1, 1]$.

b) $f'(x) = \begin{cases} -2\cos x, & x \leq 0 \\ -4e^{-4x}, & x > 0 \end{cases}$

$f'(x) = -3$ @ $\frac{\ln 3/4}{-4} = x$

$-4e^{-4x} = -3$

$e^{-4x} = 3/4$

$$\frac{\int_{-1}^{0} (1 - 2\sin x)\,dx + \int_0^1 e^{-4x}\,dx}{2} = \frac{13}{8} - \cos(-1) - \frac{1}{8}e^{-4}$$

AP Review Derivatives name Key

1.If $f(x)=x^{\frac{3}{2}}$, then $f'(4)=$

$f' = \frac{3}{2}x^{\frac{1}{2}}$

$f'(4) = \frac{3}{2} \cdot 2$

(A) –6 (B) –3 (C) 3

(D) 6 (E) 8

2. If $x^3+3xy+2y^3=17$, then in terms of x and y, $\frac{dy}{dx}=$

$3x^2 + 3x \cdot \frac{dy}{dx} + 3y + 6y^2\frac{dy}{dx} = 0$

$3x\frac{dy}{dx} + 6y^2\frac{dy}{dx} = -3x^2-3y$

$\frac{dy}{dx}(3x+6y^2) = -3x^2-3y$

$\frac{dy}{dx} = \frac{-3x^2-3y}{3x+6y^2}$

$= \frac{-\cancel{3}(x^2+y)}{\cancel{3}(x+2y^2)}$

(A) $-\frac{x^2+y}{x+2y^2}$ (B) $-\frac{x^2+y}{x+y^2}$ (C) $-\frac{x^2+y}{x+2y}$

(D) $-\frac{x^2+y}{2y^2}$ (E) $\frac{-x^2}{1+2y^2}$

3.An equation of the line tangent to the graph of $y=\frac{2x+3}{3x-2}$ at the point (1, 5) is

(A) $13x-y=8$ (B) $13x+y=18$ (C) $x-13y=64$

(D) $x+13y=66$ (E) $-2x+3y=13$

$y-5=-13(x-1)$

$y=-13x+18$

$y' = \frac{(3x-2)(2)-(2x+3)(3)}{(3x-2)^2}$

$y' = \frac{6x-4-6x-9}{(3x-2)^2} = \frac{-13}{(3x-2)^2}$

$y'(1) = -13$

4. If $y=\tan x-\cot x$, then $\frac{dy}{dx}=$

(A) $\sec x\csc x$ (B) $\sec x-\csc x$ (C) $\sec x+\csc x$

(D) $\sec^2 x-\csc^2 x$ (E) $\sec^2 x+\csc^2 x$

$y' = \sec^2 x + \csc^2 x$

5. If $f(x)=(x-1)^2\sin x$, then $f'(0)=$

(A) –2 (B) –1 (C) 0

(D) 1 (E) 2

$(x-1)^2 \cdot \cos x + \sin x \cdot 2(x-1) \cdot 1$

$(-1)^2 \cdot \cos 0 \quad + 2\cancel{(0)(0-1)} \cdot 1$

6. . The slope of the line normal to the graph of $y = 2\ln(\sec x)$ at $x = \frac{\pi}{4}$ is

$y' = 2\left(\frac{\sec x \cdot \tan x}{\sec x}\right)$

(A) -2 (B) $-\frac{1}{2}$ (C) $\frac{1}{2}$

$y' = 2\left(\frac{\cancel{\sec \frac{\pi}{4}} \cdot \tan\frac{\pi}{4}}{\cancel{\sec \frac{\pi}{4}}}\right)$

(D) 2 (E) nonexistent

$y' = 2(1) = 2$

7. If $f(x) = \left(x^2 - 2x - 1\right)^{\frac{2}{3}}$, then $f'(0)$ is

(A) $\frac{4}{3}$ (B) 0 (C) $-\frac{2}{3}$

$f' = \frac{2}{3}\left(x^2 - 2x - 1\right)^{-1/3} \cdot (2x - 2)$

(D) $-\frac{4}{3}$ (E) -2

$f'(0) = \frac{2}{3}\,(-1)\,(0-2)$

8. $\frac{d}{dx}\left(2^x\right) =$

$2^x \cdot \ln 2 \cdot 1$

(A) 2^{x-1} (B) $\left(2^{x-1}\right)x$ (C) $\left(2^x\right)\ln 2$

(D) $\left(2^{x-1}\right)\ln 2$ (E) $\frac{2x}{\ln 2}$

9.. If $f(x) = e^{3\ln(x^2)}$, then $f'(x) =$

(A) $e^{3\ln(x^2)}$ (B) $\frac{3}{x^2}e^{3\ln(x^2)}$ (C) $6(\ln x)e^{3\ln(x^2)}$

(D) $5x^4$ (E) $6x^5$

$f(x) = \cancel{e^{\ln}} x^6$

$f' = 6x^5$

10. If f is a differentiable function, then $f'(a)$ is given by which of the following?

I. $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$

II. $\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$

~~III.~~ $\lim_{x\to a}\frac{f(x+h)-f(x)}{h}$

(A) I only (B) II only (C) I and II only

(D) I and III only (E) I, II, and III

11. If $f(x)=x\sqrt{2x-3}$, then $f'(x)=$

$f' = x\cdot\frac{1}{2}(2x-3)^{-1/2}\cdot 2+\sqrt{2x-3}\cdot 1$

(A) $\frac{3x-3}{\sqrt{2x-3}}$ (B) $\frac{x}{\sqrt{2x-3}}$ (C) $\frac{1}{\sqrt{2x-3}}$

$f' = \frac{x}{\sqrt{2x-3}}+\frac{\sqrt{2x-3}}{1}$

(D) $\frac{-x+3}{\sqrt{2x-3}}$ (E) $\frac{5x-6}{2\sqrt{2x-3}}$

$f' = \frac{x+(2x-3)}{\sqrt{2x-3}}$

12. If $f(x)=-x^3+x+\frac{1}{x}$, then $f'(-1)=$

$f' = -3x^2+1-\frac{1}{x^2}$

(A) 3 (B) 1 (C) –1

$f'(-1) = -3+1-1$

(D) –3 (E) –5

13. $\frac{d}{dx}\cos^2(x^3)=$

$[\cos(x^3)]^2$

(A) $6x^2\sin(x^3)\cos(x^3)$ (B) $6x^2\cos(x^3)$

$2[\cos(x^3)]' \cdot -\sin(x^3)\cdot 3$

(C) $\sin^2(x^3)$ (D) $-6x^2\sin(x^3)\cos(x^3)$

$-6x^2\cos x^3\sin x^3$

(E) $-2\sin(x^3)\cos(x^3)$

14. An equation of the line tangent to the graph of $y = \cos(2x)$ at $x = \frac{\pi}{4}$ is

(A) $y - 1 = -\left(x - \frac{\pi}{4}\right)$

(B) $y - 1 = -2\left(x - \frac{\pi}{4}\right)$

(C) $y = 2\left(x - \frac{\pi}{4}\right)$

(D) $y = -\left(x - \frac{\pi}{4}\right)$

(E) $y = -2\left(x - \frac{\pi}{4}\right)$

$y' = -2 \sin 2x$

$y'\left(\frac{\pi}{4}\right) = -2 \sin \frac{\pi}{2}$

$= -2$

$y - \quad = -2\left(x - \frac{\pi}{4}\right)$

$y\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{2}\right) = 0$

15. At what point on the graph of $y = \frac{1}{2}x^2$ is the tangent line parallel to the line $2x - 4y = 3$?

(A) $\left(\frac{1}{2}, -\frac{1}{2}\right)$

(B) $\left(\frac{1}{2}, \frac{1}{8}\right)$

(C) $\left(1, -\frac{1}{4}\right)$

(D) $\left(1, \frac{1}{2}\right)$

(E) $(2,2)$

$2x - 4y = 3$

$-4y = -2x + 3$

$y = \frac{1}{2}x - \frac{3}{4}$

$m = \frac{1}{2}$

$y' = x$

$x = \frac{1}{2}$

$y\left(\frac{1}{2}\right) = \frac{1}{2}\left(\frac{1}{4}\right) = \frac{1}{8}$

16. If $x^2 + y^2 = 25$, what is the value of $\frac{d^2y}{dx^2}$ at the point (4, 3)?

(A) $-\frac{25}{27}$

(B) $-\frac{7}{27}$

(C) $\frac{7}{27}$

(D) $\frac{3}{4}$

(E) $\frac{25}{27}$

$2x + 2y \cdot \frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-x}{y}$

$\frac{d^2y}{dx^2} = \frac{y(-1) - (-x)\frac{dy}{dx}}{y^2}$

$\left.\frac{d^2y}{dx^2}\right|_{(4,3)} = \frac{-3 + 4\left(-\frac{4}{3}\right)}{9}$

$= \left(-3 - \frac{16}{3}\right) \cdot \frac{1}{9}$

$= -\frac{9}{27} - \frac{16}{27}$

$= -\frac{25}{27}$

17. If $f(x) = \ln|x^2 - 1|$, then $f'(x) =$

(A) $\left|\frac{2x}{x^2 - 1}\right|$

(B) $\frac{2x}{|x^2 - 1|}$

(C) $\frac{2|x|}{x^2 - 1}$

(D) $\frac{2x}{x^2 - 1}$

(E) $\frac{1}{x^2 - 1}$

$\frac{du}{u}$

*18. If $f(x) = \dfrac{e^{2x}}{2x}$, then $f'(x) =$

$f' = \dfrac{2x \cdot 2e^{2x} - e^{2x} \cdot 2}{4x^2}$

(A) 1 (B) $\dfrac{e^{2x}(1-2x)}{2x^2}$ (C) e^{2x}

$f' = \dfrac{4xe^{2x} - 2e^{2x}}{4x^2}$

(D) $\dfrac{e^{2x}(2x+1)}{x^2}$ (E) $\dfrac{e^{2x}(2x-1)}{2x^2}$

$f' = \dfrac{2e^{2x}(2x-1)}{4x^2}$ 2

*19. Let f be a function such that $\lim\limits_{h \to 0} \dfrac{f(2+h) - f(2)}{h} = 5$. Which of the following must be true?

✓I. f is continuous at $x = 2$.
✓II. f is differentiable at $x = 2$.
III. The derivative of f is continuous at $x = 2$.

(A) I only (B) II only (C) I and II only

(D) I and III only (E) II and III only

*20. Let f be the function given by $f(x) = 2e^{4x^2}$. For what value of x is the slope of the line tangent to the graph of f at $(x, f(x))$ equal to 3?

$f' = 2e^{4x^2} \cdot 8x$

(A) 0.168 (B) 0.276 (C) 0.318

$f' = 16xe^{4x^2}$

(D) 0.342 (E) 0.551

$f' = 3$ @

*21. Let $f(x) = \sqrt{x}$. If the rate of change of f at $x = c$ is twice its rate of change at $x = 1$, then $c =$

$\frac{1}{2}c^{-1/2} = 1$ $c^{-1/2} = 2$ $f' = \frac{1}{2}x^{-1/2}$

$\frac{1}{\sqrt{c}} = 2$

(A) $\dfrac{1}{4}$ (B) 1 (C) 4

$\sqrt{c} = \frac{1}{2}$

$f'(1) = \frac{1}{2}$

$f'(c) = 2(\frac{1}{2})$

(D) $\dfrac{1}{\sqrt{2}}$ (E) $\dfrac{1}{2\sqrt{2}}$

$f'(c) = 1$

22. If $x^2 + xy = 10$, then when $x = 2$, $\dfrac{dy}{dx} =$

(A) $-\dfrac{7}{2}$ (B) -2 (C) $\dfrac{2}{7}$

(D) $\dfrac{3}{2}$ (E) $\dfrac{7}{2}$

$2x + x \cdot \frac{dy}{dx} + y = 0$

$4 + 2y = 10$

$y = 3$

$\left.\frac{dy}{dx}\right|_{(2,3)} = \frac{-4-3}{2}$

$x\frac{dy}{dx} = -2x - y$

$\frac{dy}{dx} = \frac{-2x-y}{x}$

23. What is the instantaneous rate of change at $x=2$ of the function f given by $f(x)=\dfrac{x^2-2}{x-1}$?

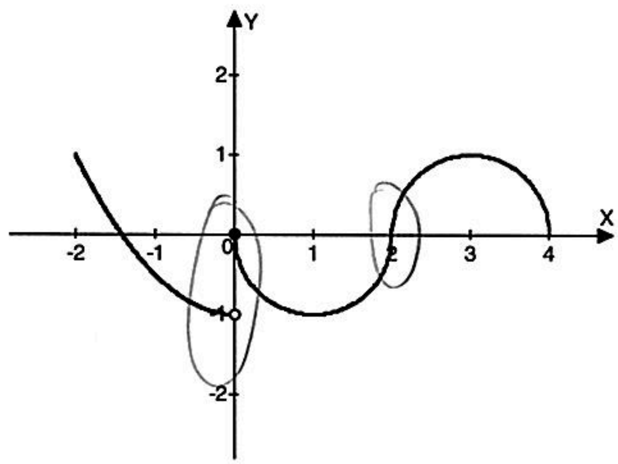
$f' = \dfrac{(x-1)\cdot 2x-(x^2-2)\cdot 1}{(x-1)^2}$

(A) -2 (B) $\frac{1}{6}$ (C) $\frac{1}{2}$

(D) 2 (E) 6

$f'(2) = \dfrac{(2-1)\cdot 4-(4-2)}{1}$

$= \dfrac{4-2}{1}$



24. The graph of the function f shown in the figure above has a vertical tangent at the point (2, 0) and horizontal tangents at the points (1, –1) and (3, 1). For what values of x, $-2<x<4$, is f not differentiable?

(A) 0 only (B) 0 and 2 only (C) 1 and 3 only

(D) 0, 1, and 3 only (E) 0, 1, 2, and 3

25. If $f(x)=\sin\left(e^{-x}\right)$, then $f'(x)=$

$\cos(e^{-x})\cdot -e^{-x}$

(A) $-\cos\left(e^{-x}\right)$ (B) $\cos\left(e^{-x}\right)+e^{-x}$ (C) $\cos\left(e^{-x}\right)-e^{-x}$

(D) $e^{-x}\cos\left(e^{-x}\right)$ (E) $-e^{-x}\cos\left(e^{-x}\right)$

26. An equation of the line tangent to the graph of $y=x+\cos x$ at the point (0, 1) is

(A) $y=2x+1$ (B) $y=x+1$ (C) $y=x$

(D) $y=x-1$ (E) $y=0$

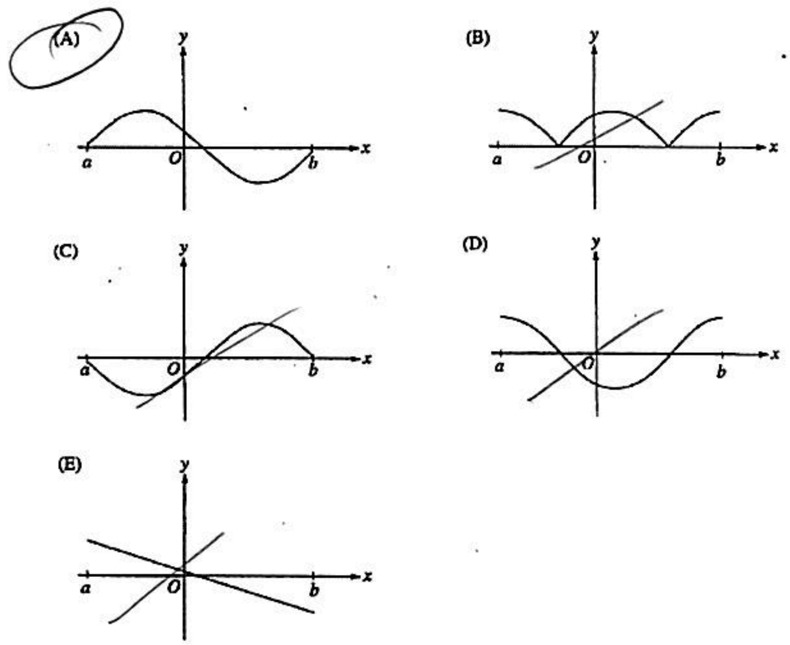
$y' = 1-\sin x$

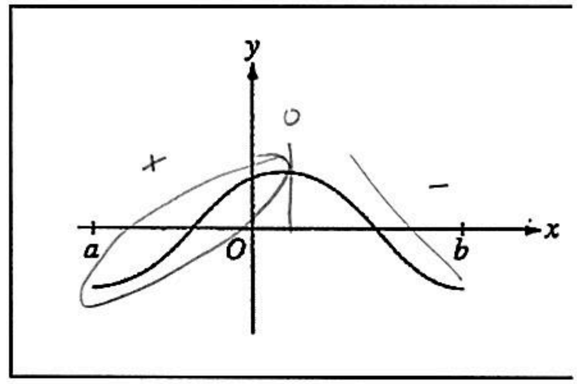
$y'(0) = 1-\sin 0 = 1$

$y-1 = 1(x-0)$

$y = x+1$

27. The graph of f is shown in the figure below. Which of the following could be the graph of the derivative of f?





x	0	1	2
$f(x)$	1	k	2

OMIT

34. The function f is continuous on the closed interval [0, 2] and has values that are given in the table above. The equation $f(x) = \frac{1}{2}$ must have at least two solutions in the interval [0, 2] if $k =$

(A) 0 (B) $\frac{1}{2}$ (C) 1

(D) 2 (E) 3

35. If $f(x) = \tan(2x)$, then $f'\left(\frac{\pi}{6}\right) =$

$2\sec^2(2x) = f'$

(A) $\sqrt{3}$ (B) $2\sqrt{3}$ (C) 4

$f'\left(\frac{\pi}{6}\right) = 2\left[\sec\frac{\pi}{3}\right]^2$

(D) $4\sqrt{3}$ (E) 8

$= 2(4)$

*36. Let f be the function given by $f(x) = 3e^{2x}$ and let g be the function given by $g(x) = 6x^3$. At what value of x do the graphs of f and g have parallel tangent lines?

(A) -0.701 (B) -0.567 (C) -0.391

(D) -0.302 (E) -0.258

$f' = 6e^{2x}$ $g' = 18x^2$

$6e^{2x} = 18x^2$

*37. If $a \neq 0$, then $\lim_{x \to a} \frac{x^2 - a^2}{x^4 - a^4}$ is

(A) $\frac{1}{a^2}$ (B) $\frac{1}{2a^2}$ (C) $\frac{1}{6a^2}$

(D) 0 (E) nonexistent

*38. Which of the following is an equation of the line tangent to the graph of $f(x) = x^4 + 2x^2$ at the point where $f'(x) = 1$?

(A) $y = 8x - 5$ (B) $y = x + 7$ (C) $y = x + 0.763$

(D) $y = x - 0.122$ (E) $y = x - 2.146$

$f' = 4x^3 + 4x$

$1 = 4x^3 + 4x$

$x = .236$

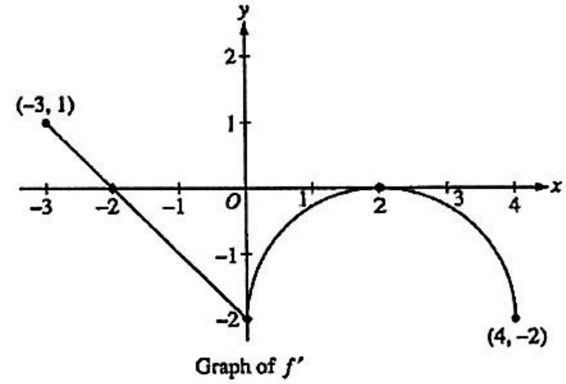
$y - .115 = 1(x - .236)$

$y = x$

Derivative Applications
AP Practice

Key

2003 AB 4 and BC 4



Graph of f'

Let f be a function defined on the closed interval $-3 \le x \le 4$ with $f(0) = 3$. The graph of f', the derivative of f, consists of one line segment and a semicircle, as shown above.

(a) On what intervals, if any, is f increasing? Justify your answer.

$(-3, -2)$ $f' > 0$ on this interval

(b) Find the x-coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$. Justify your answer.

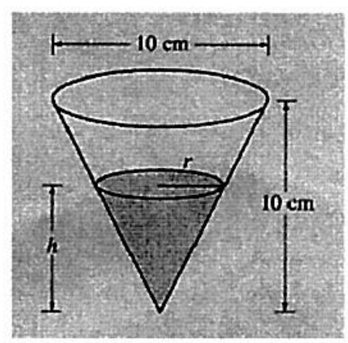
POI @ $x = 0, 2$ the slope of f', which is f'', changes signs at these x-values.

(c) Find an equation for the line tangent to the graph of f at the point $(0,3)$. $f'(0) = -2$

$y - 3 = -2(x - 0)$

2002 AB 5

A container has the shape of an open right circular cone, as shown in the figure below. The height of the container is 10 cm and the diameter of the opening is 10 cm. Water in the container is evaporating so that its depth h is changing at the constant rate of $-\frac{3}{10}$ cm/hr.



(Note: the volume of a cone of height h and radius r is given by $V = \frac{1}{3}\pi r^2 h$.)

(a) Find the volume V in the container when $h = 5$ cm. Indicate units of measure.

$V = \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 \cdot h$ $V(5) = \frac{1}{12}\pi(5)^3 \text{ cm}^3$

$\frac{r}{h} = \frac{5}{10}$

$r = \frac{1}{2}h$

(b) Find the rate of change of the volume of water in the container, with respect to time, when $h = 5$ cm. Indicate units of measure.

$V = \frac{1}{12}\pi h^3$ $\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$ $\left.\frac{dV}{dt}\right|_{h=5} = \frac{1}{4}\pi(25)\left(-\frac{3}{10}\right) \text{ cm}^3/\text{hr}$

(c) Show that the rate of change of the volume of water in the container due to evaporation is directly proportional to the exposed surface area of the water. What is the constant of proportionality?

$\frac{dV}{dt} = K\pi r^2$

$\frac{1}{4}\pi h^2 \frac{dh}{dt} = K\pi\left(\frac{1}{2}h\right)^2$

$\frac{1}{4}\pi h^2 \frac{dh}{dt} = K \cdot \frac{1}{4}\pi h^2$

$K = \frac{dh}{dt}$ which is $-\frac{3}{10}$ cm/hr

1984 AB 4 and BC 3

A function f is continuous on the closed interval $[-3,3]$ such that $f(-3)=4$ and $f(3)=1$. The function f' and f'' have the properties given in the table below.

x	$-3<x<-1$	$x=-1$	$-1<x<1$	$x=1$	$1<x<3$
$f'(x)$	Positive	Fails to exist	Negative	0	Negative
$f''(x)$	Positive	Fails to exits	Positive	0	Negative

(a) What are the x-coordinates of all absolute maximum and absolute minimum points of f on the interval $[-3,3]$? Justify your answer.

x	f
-3	4
-1	Rel max
1	no sign change
3	1

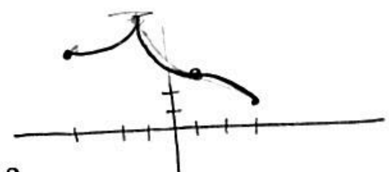
Absolute Max @ $x=-1$

Absolute Min @ $x=3$

(b) What are the x-coordinates of all points of inflection of f on the interval $[-3,3]$? Justify your answer.

POI @ $x=1$ $\quad f''$ changes sign @ $x=1$

(c) Sketch a graph that satisfies the given properties of f.



1.What is the x-coordinate of the point of inflection on the graph of $y=\frac{1}{3}x^3+5x^2+24$?

$y' = x^2+10x$

$y'' = 2x+10$

PPOI @ $x=-5$

$-$ $\quad$ $+$

-5

(A) 5 (B) 0 (C) $-\frac{10}{3}$

(D) -5 (E) -10

2.A particle moves along the x-axis so that its position at time t is given by $x(t)=t^2-6t+5$. For what value of t is the velocity of the particle zero?

$v(t) = x'(t)$

$x'(t) = 2t-6$

$0 = 2x-6$

(A) 1 (B) 2 (C) 3

(D) 4 (E) 5

3.The maximum acceleration attained on the interval $0\le t\le 3$ by the particle whose velocity is given by $v(t)=t^3-3t^2+12t+4$ is

$a(t) = v'(t)$

$v'(t) = 3t^2-6t+12$

$a'(t) = 6t-6$

$t=1$

$-$ $\quad$ $+$

0 $\quad$ 1 $\quad$ 3

(A) 9 (B) 12 (C) 14

(D) 21 (E) 40

$a(3) = 3(9)-18+12$

$= 27-18+12$

t	a(t)
0	12
1	Rel min
3	21

* 4. The graph of a function f is shown below. Which of the following statements about f is false?

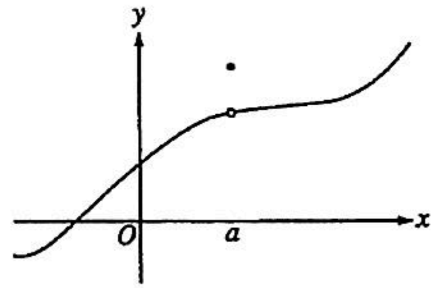
(A) f is continuous at $x = a$.

(B) f has a relative maximum at $x = a$.

(C) $x = a$ is in the domain of f.

(D) $\lim_{x\to a^+} f(x)$ is equal to $\lim_{x\to a^-} f(x)$.

(E) $\lim_{x\to a} f(x)$ exists



* 5. The radius of a circle is decreasing at a constant rate of 0.1 centimeter per second. In terms of the circumference C, what is the rate of change of the area of the circle, in square centimeters per second?

(A) $-(0.2)\pi C$

(B) $-(0.1)C$

(C) $-\frac{(0.1)C}{2\pi}$

(D) $(0.1)^2 C$

(E) $(0.1)^2 \pi C$

$A = \pi r^2$

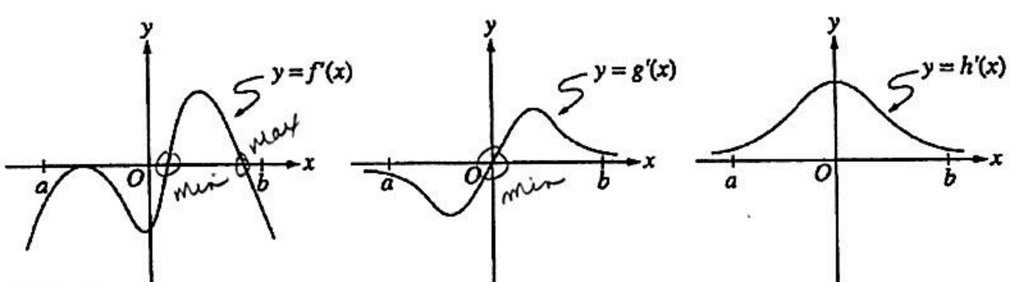
$C = 2\pi r$

$\frac{dr}{dt} = -.1$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = C(-.1)$

* 6. The graphs of the derivatives of the functions f, g, and h are shown below. Which of the functions f, g, or h have a relative maximum on the open interval $a < x < b$?



(A) f only

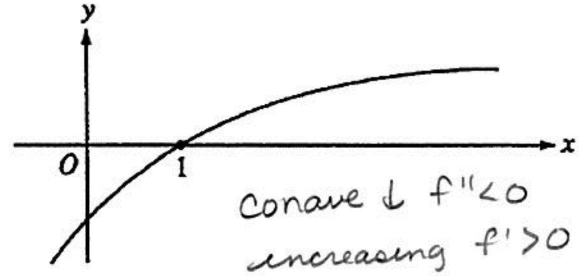
(B) g only

(C) h only

(D) f and g only

(E) f, g, and h

7. The graph of a twice-differentiable function f is shown in the figure above. Which of the following is true?



conave ↓ $f'' < 0$
increasing $f' > 0$
$f(1) = 0$

(A) $f(1) < f'(1) < f''(1)$ (B) $f(1) < f''(1) < f'(1)$

(C) $f'(1) < f(1) < f''(1)$ (D) $f''(1) < f(1) < f'(1)$ (E) $f''(1) < f'(1) < f(1)$

8. If $f''(x) = x(x+1)(x-2)^2$, then the graph of f has inflection points when $x =$

(A) -1 only (B) 2 only (C) -1 and 0 only

(D) -1 and 2 only (E) -1, 0, and 2 only

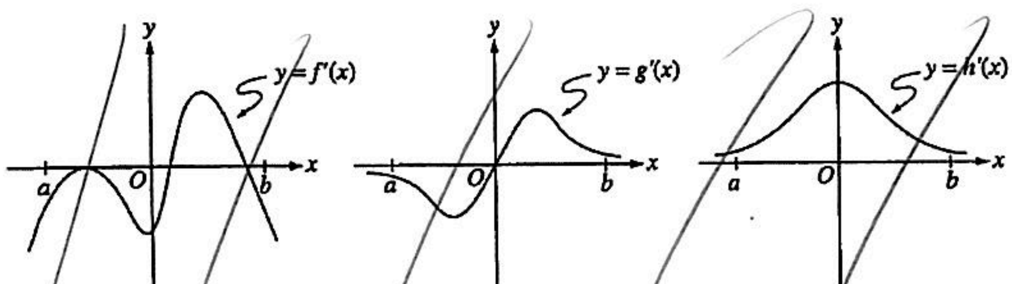
\+ − + +
−1 0 2

9. The function f is given by $f(x) = x^4 + x^2 - 2$. On which of the following intervals is f increasing?

(A) $\left(-\frac{1}{\sqrt{2}}, \infty\right)$ (B) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ (C) $(0, \infty)$

(D) $(-\infty, 0)$ (E) $\left(-\infty, -\frac{1}{\sqrt{2}}\right)$

$f' = 4x^3 + 2x$
CN@ $x = 0$ imag.
$0 = 2x(2x^2+1)$
− +
0



*10. The graphs of the derivatives of the functions f, g, and h are shown above. Which of the functions f, g, or h have a relative maximum on the open interval $a < x < b$?

(A) f only (B) g only (C) h only

(D) f and g only (E) f, g, and h

omit
Repeat

11. The first derivative of the function f is given by $f'(x)=\frac{\cos^2 x}{x}-\frac{1}{5}$. How many critical values does f have on the open interval (0, 10)?

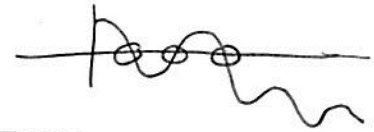
(A) One

(B) Three

(C) Four

(D) Five

(E) Seven



$f' = \frac{5\cos^2 x - x}{5x} = 0$

$5\cos^2 x - x = 0$

12. If g is a differentiable function such that $g(x)<0$ for all real numbers x and if $f'(x)=(x^2-4)g(x)$, which of the following is true?

(A) f has a relative maximum at $x=-2$ and a relative minimum at $x=2$.

(B) f has a relative minimum at $x=-2$ and a relative maximum at $x=2$.

(C) f has relative minima at $x=-2$ and at $x=2$.

(D) f has relative maxima at $x=-2$ and at $x=2$.

(E) It cannot be determined if f has any relative extrema.

CN @ $x = 2, -2$

$g(x) \neq 0$

$g(x) < 0$

− + −

-2 2

min max

13. The graph of f', the derivative of the function f, is shown above. Which of the following statements is true about f?

(A) f is decreasing for $-1 \le x \le 1$.

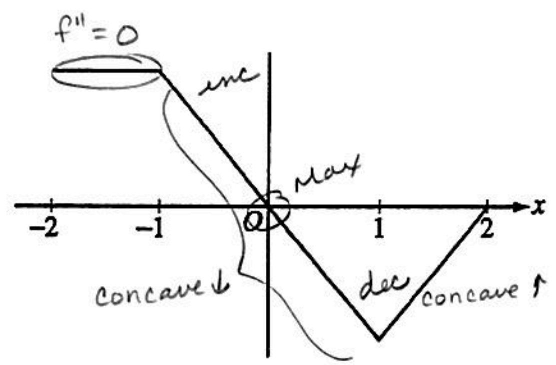
(B) f is increasing for $-2 \le x \le 0$. $f' > 0$

(C) f is increasing for $1 \le x \le 2$.

(D) f has a local minimum at $x=0$.

(E) f is not differentiable at $x=-1$ and $x=1$.

↳ f'

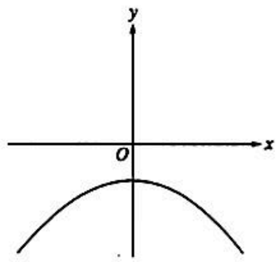


Graph of f'

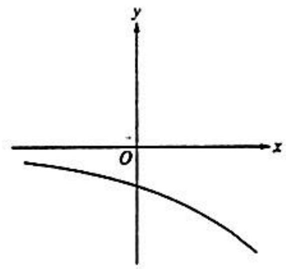
14. The function f has the property that $f(x), f'(x)$, and $f''(x)$ are negative for all real values x. Which of the following could be the graph of f?

below x-axis
decreasing
concave ↓

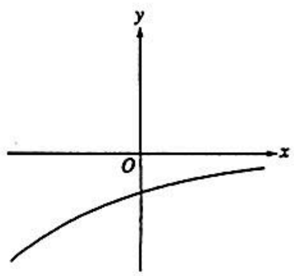
(A)



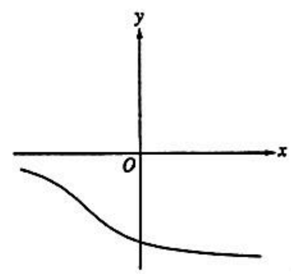
(B)



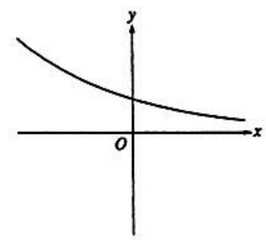
(C)



(D)



(E)



15. Let f be the function with derivative given by $f'(x) = x^2 - \frac{2}{x}$. On which of the following intervals is f decreasing?

(A) $(-\infty, -1]$ only (B) $(-\infty, 0)$ (C) $[-1, 0)$ only

(D) $(0, \sqrt[3]{2}]$ (E) $[\sqrt[3]{2}, \infty)$

$f' = \frac{x^3 - 2}{x} \begin{matrix} = 0 \\ = 0 \end{matrix}$

CN @ $x = 0, \sqrt[3]{2}$

$+ \quad \circ \quad - \quad \bullet \quad +$

$0 \qquad \sqrt[3]{2}$

16. Let f be the function given by $f(x) = 2xe^x$. The graph of f is concave down when

(A) $x < -2$ (B) $x > -2$ (C) $x < -1$

(D) $x > -1$ (E) $x < 0$

$f' = 2x \cdot e^x + e^x \cdot 2$

$f'' = 2x \cdot e^x + e^x \cdot 2 + 2e^x$

$f'' = 2xe^x + 4e^x$

$= 2e^x(x+2)$

$- \qquad +$

-2

zeros

x	-4	-3	-2	-1	0	1	2	3	4
$g'(x)$	2	3	0	-3	-2	-1	0	3	2

17. The derivative g' of a function g is continuous and has exactly two zeros. Selected values of g' are given in the table above. If the domain of g is the set of all real numbers, then g is decreasing on which of the following intervals?

$g' < 0$

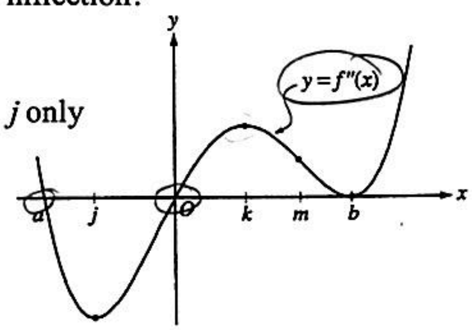
(A) $-2 \le x \le 2$ only   (B) $-1 \le x \le 1$ only   (C) $x \ge -2$

(D) $x \ge 2$ only   (E) $x \le -2$ or $x \ge 2$

18. The second derivative of the function f is given by $f''(x) = x(x-a)(x-b)^2$. The graph of f'' is shown below. For what values of x does the graph of f have a point of inflection?

(A) 0 and a only   (B) 0 and m only   (C) b and j only

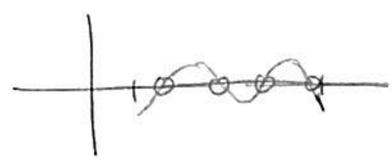
(D) 0, a, and b (E) b, j and k



*19. Let f be the function with derivative given by $f'(x) = \sin(x^2+1)$. How many relative extrema does f have on the interval $2 < x < 4$?

(A) One   (B) Two   (C) Three

(D) Four   (E) Five



*20 For all x in the closed interval $[2,5]$, the function f has a positive first derivative and a negative second derivative. Which of the following could be a table of values for f?

increasing @ decreasing rate

(A)

x	$f(x)$
2	7
3	9
4	12
5	16

>2 >3 >4 increasing rate

(B)

x	$f(x)$
2	7
3	11
4	14
5	16

>4 >3 >2 decreasing rate

(C)

x	$f(x)$
2	16
3	12
4	9
5	7

dec

(D)

x	$f(x)$
2	16
3	14
4	11
5	7

dec

(E)

x	$f(x)$
2	16
3	13
4	10
5	7

dec

*21. A railroad track and a road cross at right angles. An observer stands on the road 70 meters south of the crossing and watches an eastbound train traveling at 60 meters per second. At how many meters per second is the train moving away from the observer 4 seconds after it passes through the intersection?

(A) 57.60 (B) 57.88 (C) 59.20

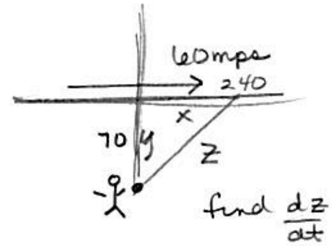
(D) 60.00 (E) 67.40

Const

$x^2 + \cancel{y^2} = z^2$

$x\frac{dx}{dt} = z\frac{dz}{dt}$

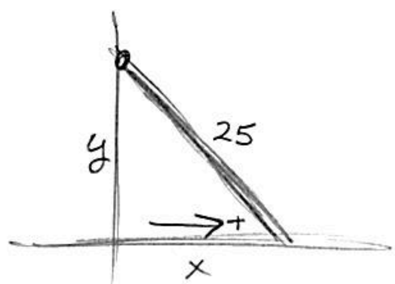
$240(60) = 250\frac{dz}{dt}$



22. The top of a 25-foot ladder is sliding down a vertical wall at a constant rate of 3 feet per minute. When the top of the ladder is 7 feet from the ground, what is the rate of change of the distance between the bottom of the ladder and the wall?

~~(A)~~ $-\frac{7}{8}$ feet per minute ~~(B)~~ $-\frac{7}{24}$ feet per minute

(C) $\frac{7}{24}$ feet per minute (D) $\frac{7}{8}$ feet per minute (E) $\frac{21}{25}$ feet per minute

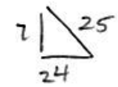


$x^2 + y^2 = 25^2$

$x\frac{dx}{dt} + y\frac{dy}{dt} = 0$

$24\frac{dx}{dt} + 7(-3) = 0$

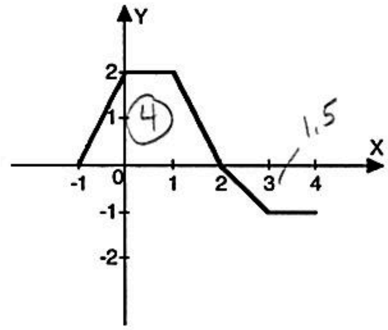
$\frac{dx}{dt} = \cancel{\frac{21}{24}}\ \frac{7}{8}$



$\frac{dy}{dt} = -3$

AP Practice – Integration name Key

1. The graph of a piecewise linear function f, for $-1 \le x \le 4$, is shown below. What is the value of $\int_{-1}^{4} f(x)\,dx$?



(A) 1 (B) 2.5 (C) 4

(D) 5.5 (E) 8

2. $\int_{1}^{2} \frac{1}{x^2}\,dx =$

(A) $-\frac{1}{2}$ (B) $\frac{7}{24}$ (C) $\frac{1}{2}$

(D) 1 E) $2\ln 2$

$\int_1^2 x^{-2}\,dx$

$-\frac{1}{x}\Big|_1^2 = -\frac{1}{2} + 1$

3. $\int_{0}^{x} \sin t \ dt =$

$-\cos x + \overset{1}{\cancel{\cos 0}}$

(A) $\sin x$ (B) $-\cos x$ (C) $\cos x$

(D) $\cos x - 1$ (E) $1 - \cos x$

4. $\int_{1}^{e} \left(\frac{x^2 - 1}{x} \right) dx =$

$\int_1^e \left(x - \frac{1}{x}\right) dx$

(A) $e - \frac{1}{e}$ (B) $e^2 - e$ (C) $\frac{e^2}{2} - e + \frac{1}{2}$

(D) $e^2 - 2$ (E) $\frac{e^2}{2} - \frac{3}{2}$

$\frac{x^2}{2} - \ln x \Big|_1^e$

$\frac{e^2}{2} - 1 - \left(\frac{1}{2} - 0\right)$

$\frac{e^2}{2} - \frac{3}{2}$

5. If f is a linear function and $0 < a < b$, then $\int_{a}^{b} f''(x)\,dx =$

(A) 0 (B) 1 (C) $\frac{ab}{2}$

(D) $b - a$ (E) $\frac{b^2 - a^2}{2}$

$f'(b) - f'(a)$

f' is a constant,
so $f'(b) = f'(a)$

6. If $F(x)=\int_0^x \sqrt{t^3+1}\,dt$, then $F'(2)=$

FTC Part 2

(A) -3 (B) -2 (C) 2

(D) 3 (E) 18

$\left.\sqrt{x^3+1}\cdot 1\right|_{x=2}$

$\sqrt{9}$

7. What are all values of k for which $\int_{-3}^{k} x^2\,dx=0$?

(A) -3 (B) 0 (C) 3

(D) -3 and 3 (E) -3, 0, and 3

$\left.\frac{x^3}{3}\right|_{-3}^{k} = \frac{k^3}{3}+9=0$

$\frac{k^3}{3}=-9$

$k^3=-27$

$k=-3$

8. If $\frac{dy}{dt}=ky$ and k is a nonzero constant, then y could be

(A) $2e^{kty}$ (B) $2e^{kt}$ (C) $e^{kt}+3$

(D) $kty+5$ (E) $\frac{1}{2}ky^2+\frac{1}{2}$

$\int \frac{1}{y}\,dy = \int k\,dt$

$\ln y = kt+c$

$e^{kt+c}=y$

9. What is the average value of $y=x^2\sqrt{x^3+1}$ on the interval [0, 2]?

(A) $\frac{26}{9}$ (B) $\frac{52}{9}$ (C) $\frac{26}{3}$

(D) $\frac{52}{3}$ (E) 24

$\frac{\int_0^2 x^2\sqrt{x^3+1}\,dx}{2}$

*10. If f is a continuous function and if $F'(x)=f(x)$ for all real numbers x, then $\int_1^3 f(2x)\,dx=$

(A) $2F(3)-2F(1)$ (B) $\frac{1}{2}F(3)-\frac{1}{2}F(1)$ (C) $2F(6)-2F(2)$

(D) $F(6)-F(2)$ (E) $\frac{1}{2}F(6)-\frac{1}{2}F(2)$

$F' = \frac{1}{2}$

x	2	5	7	8
$f(x)$	10	30	40	20

*11. The function f is continuous on the closed interval $[2, 8]$ and has values that are given in the table above. Using the subintervals $[2, 5], [5, 7], [7, 8]$, what is the trapezoidal approximation of $\int_2^8 f(x)\,dx$

$\frac{10+30}{2}\cdot 3 + \frac{30+40}{2}\cdot 2 + \frac{40+20}{2}\cdot 1$

(A) 110 (B) 130 (C) 160 $60 + 70 + 30$

(D) 190 (E) 210

*12. Let $F(x)$ be an antiderivative of $\frac{(\ln x)^3}{x}$. If $F(1) = 0$, then $F(9) =$

$u = \ln x$

$du = \frac{1}{x}dx$

(A) 0.048 (B) 0.144 (C) 5.827

$dx = x\,du$

(D) 23.308 (E) 1,640.250

$\int \frac{u^3}{\cancel{x}} \cdot \cancel{x}\,du$

$\frac{(\ln 9)^4}{4}$

$\frac{u^4}{4} + C = F(u)$

13. $\int_0^1 e^{-4x}\,dx =$

(A) $\frac{-e^{-4}}{4}$ (B) $-4e^{-4}$ (C) $e^{-4} - 1$

$\frac{(\ln 1)^4}{4} + C = 0$

(D) $\frac{1}{4} - \frac{e^{-4}}{4}$ (E) $4 - 4e^{-4}$

$-\frac{1}{4}e^{-4x}\Big|_0^1$

$C = 0$

$-\frac{1}{4}e^{-4} + \frac{1}{4}\cancel{e^{0}}$

14. $\int_0^{\frac{\pi}{4}} \sin x\,dx =$

$-\cos x\Big|_0^{\pi/4}$

(A) $-\frac{\sqrt{2}}{2}$ (B) $\frac{\sqrt{2}}{2}$ (C) $-\frac{\sqrt{2}}{2} - 1$

$-\cos \pi/4 + \cos 0$

(D) $-\frac{\sqrt{2}}{2} + 1$ (E) $\frac{\sqrt{2}}{2} - 1$

$-\frac{\sqrt{2}}{2} + 1$

15. $\int x^2 \cos(x^3)\,dx =$

$u = x^3$

(A) $-\frac{1}{3}\sin(x^3) + C$ (B) $\frac{1}{3}\sin(x^3) + C$ (C) $-\frac{x^3}{3}\sin(x^3) + C$

$du = 3x^2\,dx$

(D) $\frac{x^3}{3}\sin(x^3) + C$ (E) $\frac{x^3}{3}\sin\left(\frac{x^4}{4}\right) + C$

$\frac{1}{3}\int \cos u\,du$

$\frac{1}{3}\sin(x^3) + C$

16. Using the substitution $u = 2x+1$, $\int_0^2 \sqrt{2x+1}\,dx$ is equivalent to

$u = 2x-1$
$du = 2dx$

(A) $\frac{1}{2}\int_{-1/2}^{1/2} \sqrt{u}\,du$ (B) $\frac{1}{2}\int_0^2 \sqrt{u}\,du$ (C) $\frac{1}{2}\int_1^5 \sqrt{u}\,du$

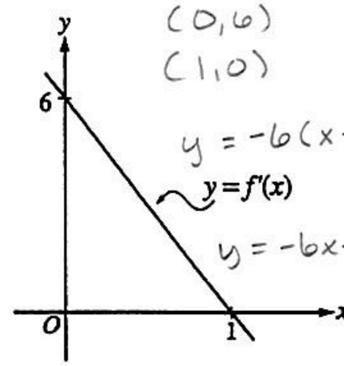
(D) $\int_0^2 \sqrt{u}\,du$ (E) $\int_1^5 \sqrt{u}\,du$

$\frac{1}{2}\int_1^5 u^{1/2}\,du$

17. The graph of f', the derivative of f, is the line shown in the figure above. If $f(0)=5$, then $f(1)=$

(A) 0 (B) 3 (C) 6

(D) 8 (E) 11



(0,6)
(1,0)
$y = -6(x-1)$
$y = -6x+6$

$\int(-6x+6)\,dx$
$-\frac{6x^2}{2} + 6x + C$
$-3x^2+6x+C = f$
$C = 5$

$f = -3x^2+6x+5$
$f(1) = -3+6+5$

18. $\frac{d}{dx}\left(\int_0^{x^2} \sin(t^3)\,dt\right) =$

$\sin x^6 \cdot 2x$

FTC

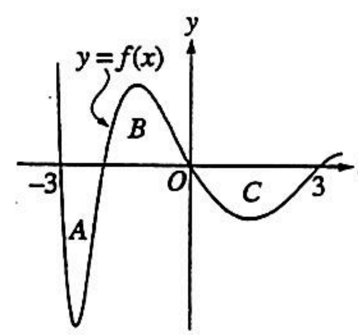
(A) $-\cos(x^6)$ (B) $\sin(x^3)$ C) $\sin(x^6)$

(D) $2x\sin(x^3)$ (E) $2x\sin(x^6)$

19. The regions A, B and C in the figure above are bounded by the graph of the function f and the x-axis. If the area of each region is 2, what is the value of $\int_{-3}^{3}(f(x)+1)\,dx$?

(A) -2 (B) -1 (C) 4

(D) 7 (E) 12

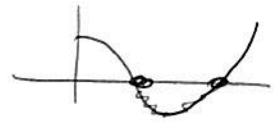


$-\cancel{2}+\cancel{2}-2+6$

*20. The rate of change of the altitude of a hot-air balloon is given by $r(t) = t^3 - 4t^2 + 6$ for $0 \le t \le 8$. Which of the following expressions gives the change in altitude of the balloon during the time the altitude is decreasing?

(A) $\int_{1.572}^{3.514} r(t)\,dt$ (B) $\int_0^8 r(t)\,dt$ (C) $\int_0^{2.667} r(t)\,dt$

(D) $\cancel{\int_{1.572}^{3.514} r'(t)\,dt}$ (E) $\cancel{\int_0^{2.667} r'(t)\,dt}$



$r<0$

$\int r(t)\,dt$

*21. The velocity, in ft/sec, of a particle moving along the x-axis is given by the function $v(t) = e^t + te^t$. What is the average velocity of the particle from time $t = 0$ to time $t = 3$?

(A) 20.086 ft/sec (B) 26.447 ft/sec (C) 32.809 ft/sec

(D) 40.671 ft/sec (E) 79.342 ft/sec

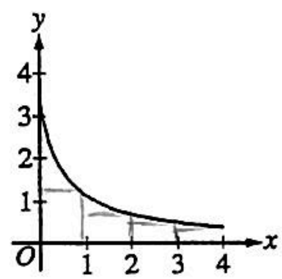
$\frac{\int_0^3 v(t)\,dt}{3}$

concave ↑

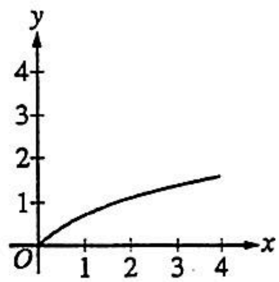
*22. If a trapezoidal sum overapproximates $\int_0^4 f(x)\,dx$, and a right Riemann sum underapproximates $\int_0^4 f(x)\,dx$, which of the following could be the graph of $y = f(x)$?

decreasing

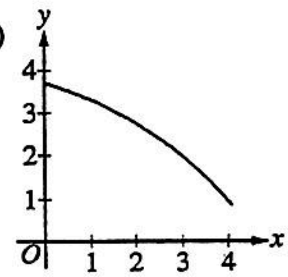
(A)



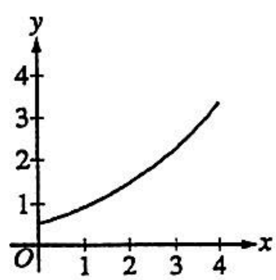
(B)



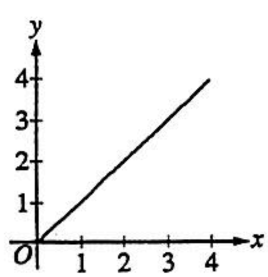
(C)



(D)

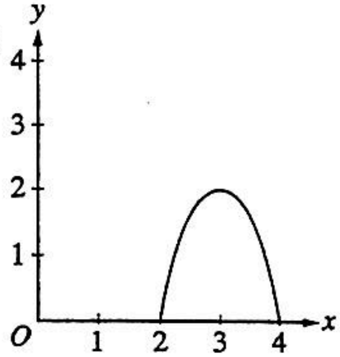


(E)

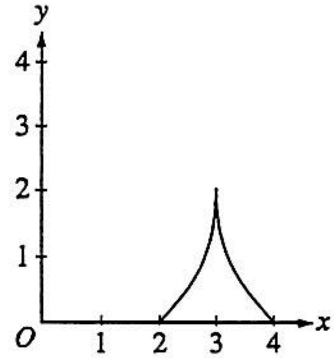


*23. On the closed interval $[2,4]$, which of the following could be the graph of a function f with the property that $\frac{1}{4-2}\int_2^4 f(t)\,dt = 1$?

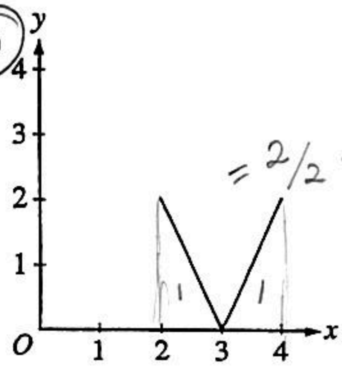
(A)



(B)

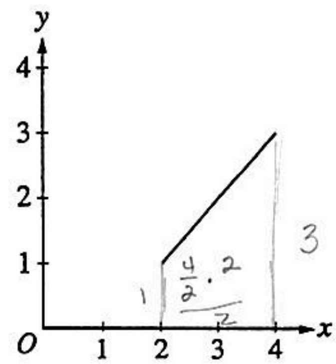


(C)



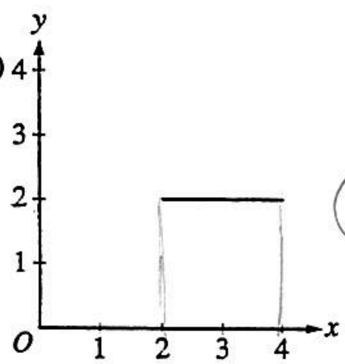
$= 2/2 = 1$

(D)



$3 \quad = 2$

(E)



$= 2$

Average value $= 1$

$\frac{\text{Area}}{\text{width}} = 1$

t (hours)	0	2	5	7	8
$E(t)$ (hundreds of entries)	0	4	13	21	23

2. A zoo sponsored a one-day contest to name a new baby elephant. Zoo visitors deposited entries in a special box between noon ($t = 0$) and 8 P.M. ($t = 8$). The number of entries in the box t hours after noon is modeled by a differentiable function E for $0 \le t \le 8$. Values of $E(t)$, in hundreds of entries, at various times t are shown in the table above.

(a) Use the data in the table to approximate the rate, in hundreds of entries per hour, at which entries were being deposited at time $t = 6$. Show the computations that lead to your answer.

(b) Use a trapezoidal sum with the four subintervals given by the table to approximate the value of $\frac{1}{8}\int_0^8 E(t)\,dt$. Using correct units, explain the meaning of $\frac{1}{8}\int_0^8 E(t)\,dt$ in terms of the number of entries.

(c) At 8 P.M., volunteers began to process the entries. They processed the entries at a rate modeled by the function P, where $P(t) = t^3 - 30t^2 + 298t - 976$ hundreds of entries per hour for $8 \le t \le 12$. According to the model, how many entries had not yet been processed by midnight ($t = 12$)?

(d) According to the model from part (c), at what time were the entries being processed most quickly? Justify your answer.

a) $\frac{21-13}{7-5}$ hundreds of entries/hour

b) $\frac{1}{8}\int_0^8 E(T)\,dt$ is the average # of entries in hundreds of entries per hour between noon & 8 pm

$$\frac{\frac{0+4}{2}\cdot 2 + \frac{4+13}{2}\cdot 3 + \frac{13+21}{2}\cdot 2 + \frac{21+23}{2}\cdot 1}{8}$$ hundred entries

c) $\int_8^{12} P(t)\,dt = 1600$ entries

$2300 - 1600 = 700$ entries left at midnight

d) $P'(t) = 0$ @ 9.1835034 & 10.816497

$P'(t) = 3t^2 - 60t + 298$

@ $t = 12$ hours or midnight

t	$P(t)$
9	0
9.183	5.088
10.816	Rel min
12	8

AP Integral Applications Practice name Key

1. The approximate value of $y = \sqrt{4+\sin x}$ at $x = 0.12$, obtained from the tangent to the graph at $x = 0$, is

(A) 2.00 (B) 2.03 (C) 2.06

(D) 2.12 (E) 2.24

$y' = \frac{1}{2}(4+\sin x)^{-1/2} \cdot \cos x$

$y'(0) = \frac{1}{2} \cdot \frac{1}{\sqrt{4+0}} \cdot 1 = \frac{1}{4}$

$y - 2 = \frac{1}{4}(x-0)$

$y - 2 = \frac{1}{4}(.12)$

2. The number of bacteria in a culture is growing at a rate of $3000e^{2t/5}$ per unit of time t. At $t = 0$, the number of bacteria present was 7,500. Find the number present at $t = 5$.

(A) $1{,}200e^2$ (B) $3{,}000e^2$ (C) $7{,}500e^2$

(D) $7{,}500e^5$ (E) $\frac{15{,}000}{7}e^7$

$7500 + \int_0^5 3000\, e^{\frac{2t}{5}}\, dt$

$7500 + \left(\frac{5}{2} \cdot 3000\, e^{\frac{2t}{5}}\right)\Big|_0^5 = \cancel{7500 + 7500e^2} \quad \cancel{-7500e^0} = 7500e^2$

? Typo?

3. The acceleration of a particle moving along the x-axis at time t is given by $a(t) = 6t - 2$. If the velocity is 25 when $t = 3$ and the position is 10 when $t = 1$, then the position $x(t) =$

(A) $9t^2 + 1$ (B) $3t^2 - 2t + 4$ (C) $t^3 - t^2 + 4t + 6$

(D) $t^3 - t^2 + 9t - 20$ (E) $36t^3 - 4t^2 - 77t + 55$

$\int a(t) = 3t^2 - 2t + C$

$v(t) = 3t^2 - 2t + C$

$25 = 27 - 6 + C$,

$C = 4$ ✓

$\int 3t^2 - 2t + 4 =$

$t^3 - t^2 + 4t + C$

$10 = 1 - 1 + 4 + C$

*4. A puppy weighs 2.0 pounds at birth and 3.5 pounds two months later. If the weight of the puppy during its first 6 months is increasing at a rate proportional to its weight, then how much will the puppy weigh when it is 3 months old?

(A) 4.2 pounds (B) 4.6 pounds (C) 4.8 pounds

(D) 5.6 pounds (E) 6.5 pounds

$\frac{dw}{dt} = kw$

$w = Ce^{kt}$

$w = 2e^{kt}$

$3.5 = 2e^{k \cdot 2}$

$1.75 = e^{2k}$ $\quad k = \frac{\ln 1.75}{2}$

*5. Let f be a differentiable function such that $f(3) = 2$ and $f'(3) = 5$. If the tangent line to the graph of f at $x = 3$ is used to find an approximation to a zero of f, that approximation is

$y = 0$

(A) 0.4 (B) 0.5 (C) 2.6

(D) 3.4 (E) 5.5

$y - 2 = 5(x - 3)$

$y - 2 = 5x - 15$

$0 - 2 = 5x - 15$

$13 = 5x$

$x = \frac{13}{5}$

*6. At time $t \geq 0$, the acceleration of a particle moving on the x-axis is $a(t) = t + \sin t$. At $t = 0$, the velocity of the particle is -2. For what value of t will the velocity of the particle be zero?

(A) 1.02 (B) 1.48 (C) 1.85

(D) 2.81 (E) 3.14

$-2 + \int a(t)\,dt = v(t)$

$\int (t + \sin t)\,dt = \frac{t^2}{2} - \cos t = 0$

*7. Population y grows according to the equation $\frac{dy}{dt} = ky$, where k is a constant and t is measured in years. If the population doubles every 10 years, then the value of k is

$2 = 1e^{k \cdot 10}$

$\frac{\ln 2}{10}$

(A) 0.069 (B) 0.200 (C) 0.301

(D) 3.322 (E) 5.000

*8. Let $F(x)$ be an antiderivative of $\frac{(\ln x)^3}{x}$. If $F(1) = 0$, then $F(9) =$

$\int \frac{(\ln x)^3}{x}\,dx = \frac{(\ln x)^4}{4} + C$

(A) 0.048 (B) 0.144 (C) 5.827

(D) 23.308 (E) 1,640.250

$0 = \frac{(\ln 1)^4}{4} + C$

$C = 0$

$\frac{(\ln 9)^4}{4} =$

*9. A pizza, heated to a temperature of 350 degrees Fahrenheit (°F), is taken out of an oven and placed in a 75°F room at time $t = 0$ minutes. The temperature of the pizza is changing at a rate of $-110e^{-0.4t}$ degrees Fahrenheit per minute. To the nearest degree, what is the temperature of the pizza at time $t = 5$ minutes?

(A) 112°F (B) 119°F (C) 147°F

$\int_0^5 -110e^{-.4t}\,dt =$

(D) 238°F (E) ~~335°F~~

*10. The base of a solid is a region in the first quadrant bounded by the x-axis, the y-axis, and the line $x + 2y = 8$, as shown in the figure below. If cross sections of the solid perpendicular to the x-axis are semicircles, what is the volume of the solid?

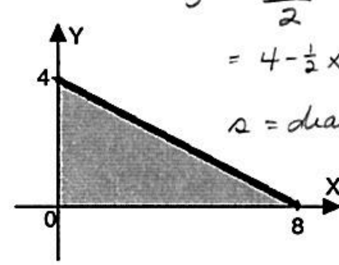
$y = \frac{8-x}{2}$

$= 4 - \frac{1}{2}x$

(A) 12.566 (B) 14.661 (C) 16.755

(D) 67.021 (E) 134.041

s = diameter

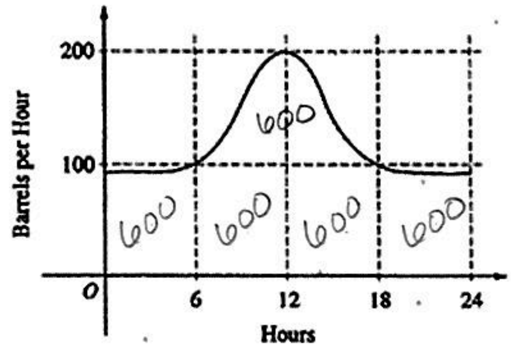


$\frac{\pi}{2}\int_0^8 \left(\frac{8-x}{4}\right)^2 dx$

11. The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph shown below. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?

(A) 500 (B)600 (C) 2,400

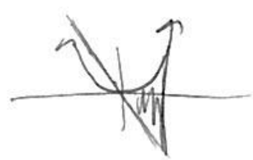
(D) 3,000 (E) 4,800



12. What is the area of the region between the graphs of $y = x^2$ and $y = -x$ from $x = 0$ to $x = 2$?

(A) $\frac{2}{3}$ (B) $\frac{8}{3}$ (C) 4

(D) $\frac{14}{3}$ (E) $\frac{16}{3}$



$\int_0^2 (x^2 + x)\,dx$

$\frac{x^3}{3} + \frac{x^2}{2}\Big|_0^2$

$\frac{8}{3} + 2 - 0$

*13. If $0 \le k < \frac{\pi}{2}$ and the area under the curve $y = \cos x$ from $x = k$ to $x = \frac{\pi}{2}$ is 0.1, then $k =$

(A) 1.471 (B) 1.414 (C) 1.277

(D) 1.120 (E) 0.436

$\int_k^{\frac{\pi}{2}} \cos x = .1$

$\sin\frac{\pi}{2} - \sin k = .1$

$\sin k = .9$

14. The rate of change of the volume, V, of water in a tank with respect to time, t, is directly proportional to the square root of the volume. Which of the following is a differential equation that describes this relationship?

(A) $V(t) = k\sqrt{t}$ (B) $V(t) = k\sqrt{V}$ (C) $\frac{dV}{dt} = k\sqrt{t}$

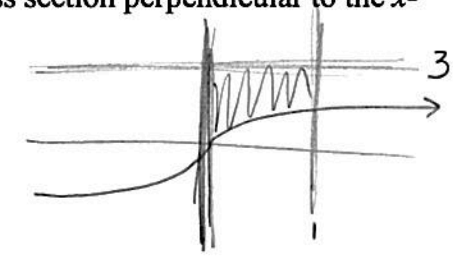
(D) $\frac{dV}{dt} = \frac{k}{\sqrt{V}}$ (E) $\frac{dV}{dt} = k\sqrt{V}$

$\frac{dv}{dt} = k\sqrt{v}$

*15. The base of a solid is the region in the first quadrant bounded by the y-axis, the graph of $y = \tan^{-1} x$, the horizontal line $y = 3$, and the vertical line $x = 1$. For this solid, each cross section perpendicular to the x-axis is a square. What is the volume of the solid?

(A) 2.561 (B) 6.612 (C) 8.046

(D) 8.755 (E) 20.773

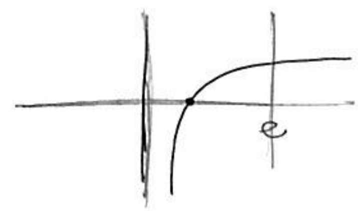


$\int_0^1 (3 - \tan^{-1} x)^2 dx$

Try to do these without a calculator.

1979 AB 5 and BC 5

Let R be the region bounded by the graph of $y = \frac{1}{x}\ln x$, the x-axis, and the line $x = e$.



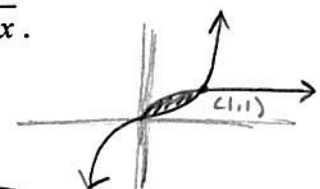
(a) Find the area of the region R. $\int_1^e \frac{\ln x}{x} dx = \left.\frac{(\ln x)^2}{2}\right|_1^e$

$\frac{(\ln e)^2}{2} - \frac{(\ln 1)^2}{2} = \frac{1}{2}$

1980 AB 1

Let R be the region enclosed by the graphs of $y = x^3$ and $y = \sqrt{x}$.

(a) Find the area of R. $\int_0^1 (\sqrt{x} - x^3) dx$



$\left.\frac{2}{3}x^{3/2} - \frac{x^4}{4}\right|_0^1 = \frac{2}{3} - \frac{1}{4}$

(b) Find the volume of the solid generated by revolving R about the x-axis.

$\pi\int_0^1 (\sqrt{x})^2 dx - \pi\int_0^1 x^6 dx$

$\pi\left(\left.\frac{x^2}{2}\right|_0^1 - \left.\frac{x^7}{7}\right|_0^1\right) = \pi\left(\frac{1}{2} - \frac{1}{7}\right)$

1981 AB 2

Let R be the region in the <u>first quadrant</u> enclosed by the graphs of $y = 4 - x^2$, $y = 3x$, and the <u>y-axis</u>.

(a) Find the area of region R. $\int_0^1 (4 - x^2 - 3x) dx$

$4 - x^2 = 3x$
$0 = x^2 + 3x - 4$
$(x+4)(x-1$

$4 - \frac{1}{3} - \frac{3}{2} - 0$

$\left.4x - \frac{x^3}{3} - \frac{3x^2}{2}\right|_0^1$

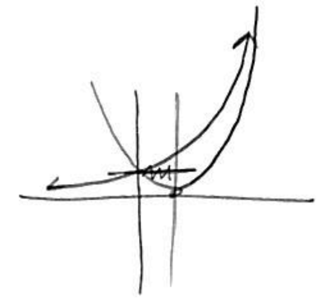
(b) Find the volume of the solid formed by revolving the region R about the <u>x-axis</u>.

$\pi\int_0^1 (4-x^2)^2 dx - \pi\int_0^1 (3x)^2 dx = \pi\int_0^1 (16 - 8x^2 + x^4 - 9x^2) dx$

$= \pi\left.\left(16x - \frac{8x^3}{3} + \frac{x^5}{5} - \frac{9x^3}{3}\right)\right|_0^1$

$= \pi(16 - 8/3 + 1/5 - 3)$

1990 AB 3



Let R be the region enclosed by the graphs of $y = e^x$, $y = (x-1)^2$, and the line $x = 1$.

(a) Find the area of R.

$$\int_0^1 [e^x - (x-1)^2]\,dx = 1.384$$

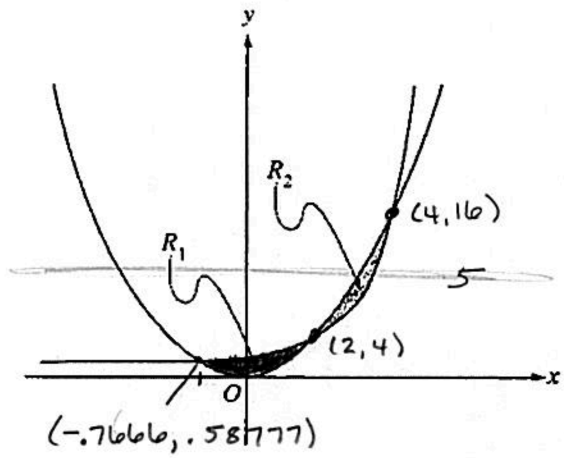
(b) Find the volume of the solid generated when R is revolved about the x-axis.

$$\pi \int_0^1 (e^{2x})\,dx - \pi \int_0^1 (x-1)^4\,dx = 9.407$$

(c) Set up, but do not integrate, an integral expression in terms of a single variable for the volume of the solid generated when R is revolved about the y-axis.

$$\pi \int_0^1 [1^2 - (1-\sqrt{y})^2]\,dy + \pi \int_1^e (1^2 - (\ln y)^2)\,dy$$

1995 AB 4 and BC 2 (Calculator)



Note: Figure not drawn to scale.

The shaded regions R_1 and R_2 shown above are enclosed by the graphs of $f(x) = x^2$ and $g(x) = 2^x$.

(a) Find the x- and y-coordinates of the three points of intersection of the graphs of f and g.

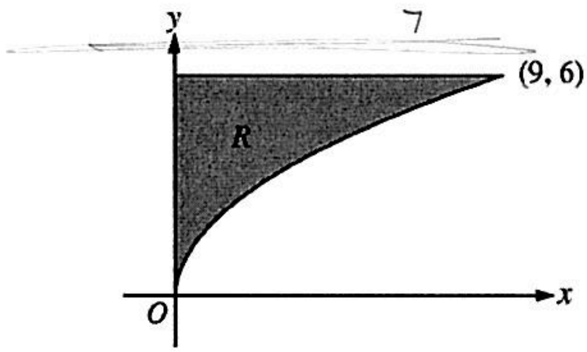
see graph

(b) Without using absolute value, set up an expression involving one or more integrals that gives the total area enclosed by the graphs of f and g. Do not evaluate.

$$\int_{-.766}^{2} (2^x - x^2)\,dx + \int_2^4 (x^2 - 2^x)\,dx$$

(c) Without using absolute value, set up an expression involving one or more integrals that gives the volume of the solid generated by revolving the region R_1 about the line $y = 5$. Do not evaluate.

$$\pi \int_{-.766}^{2} [(5-x^2)^2 - (5-2^x)^2]\,dx$$



$\left(\frac{y}{2}\right)^2 = x$

4. Let R be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the y-axis, as shown in the figure above.
   (a) Find the area of R. $\int_0^9 (6 - 2\sqrt{x})\,dx = 18$
   (b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 7$. $\pi \int_0^9 [(7-2\sqrt{x})^2 - 1]\,dx$
   (c) Region R is the base of a solid. For each y, where $0 \le y \le 6$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose height is 3 times the length of its base in region R. Write, but do not evaluate, an integral expression that gives the volume of the solid.

$\int_0^6 (s \cdot 3s)\ dy$ $\qquad s = \left(\frac{y^2}{4}\right)$

Question 5

At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function W models the total amount of solid waste stored at the landfill. Planners estimate that W will satisfy the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ for the next 20 years. W is measured in tons, and t is measured in years from the start of 2010.

(a) Use the line tangent to the graph of W at $t = 0$ to approximate the amount of solid waste that the landfill contains at the end of the first 3 months of 2010 (time $t = \frac{1}{4}$).

(b) Find $\frac{d^2W}{dt^2}$ in terms of W. Use $\frac{d^2W}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the amount of solid waste that the landfill contains at time $t = \frac{1}{4}$.

(c) Find the particular solution $W = W(t)$ to the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ with initial condition $W(0) = 1400$.

see attached

Question 5

At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function W models the total amount of solid waste stored at the landfill. Planners estimate that W will satisfy the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ for the next 20 years. W is measured in tons, and t is measured in years from the start of 2010.

(a) Use the line tangent to the graph of W at $t = 0$ to approximate the amount of solid waste that the landfill contains at the end of the first 3 months of 2010 (time $t = \frac{1}{4}$).

(b) Find $\frac{d^2W}{dt^2}$ in terms of W. Use $\frac{d^2W}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the amount of solid waste that the landfill contains at time $t = \frac{1}{4}$.

(c) Find the particular solution $W = W(t)$ to the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ with initial condition $W(0) = 1400$.

(a) $\left.\frac{dW}{dt}\right|_{t=0} = \frac{1}{25}(W(0) - 300) = \frac{1}{25}(1400 - 300) = 44$

The tangent line is $y = 1400 + 44t$.

$W\left(\frac{1}{4}\right) \approx 1400 + 44\left(\frac{1}{4}\right) = 1411$ tons

$2: \begin{cases} 1: \frac{dW}{dt} \text{ at } t = 0 \\ 1: \text{answer} \end{cases}$

(b) $\frac{d^2W}{dt^2} = \frac{1}{25}\frac{dW}{dt} = \frac{1}{625}(W - 300)$ and $W \geq 1400$

Therefore $\frac{d^2W}{dt^2} > 0$ on the interval $0 \leq t \leq \frac{1}{4}$.

The answer in part (a) is an underestimate.

$2: \begin{cases} 1: \frac{d^2W}{dt^2} \\ 1: \text{answer with reason} \end{cases}$

(c) $\frac{dW}{dt} = \frac{1}{25}(W - 300)$

$\int \frac{1}{W - 300}\, dW = \int \frac{1}{25}\, dt$

$\ln|W - 300| = \frac{1}{25}t + C$

$\ln(1400 - 300) = \frac{1}{25}(0) + C \Rightarrow \ln(1100) = C$

$W - 300 = 1100e^{\frac{1}{25}t}$

$W(t) = 300 + 1100e^{\frac{1}{25}t},\ 0 \leq t \leq 20$

$5: \begin{cases} 1: \text{separation of variables} \\ 1: \text{antiderivatives} \\ 1: \text{constant of integration} \\ 1: \text{uses initial condition} \\ 1: \text{solves for } W \end{cases}$

Note: max $2/5$ [1-1-0-0-0] if no constant of integration

Note: $0/5$ if no separation of variables