Limits and Continuity Packet name ______________________________

Note-Any problem with a * is Calculator Active

1. $\lim_{n\to\infty} \frac{3n^3 - 5n}{n^3 - 2n^2 + 1}$ is

(A) -5 (B) -2 (C) 1

(D) 3 (E) nonexistent

2.If the function *f* is continuous for all real numbers and if $f(x) = \frac{x^2 - 4}{x + 2}$ when $x \neq -2$, then $f(-2) =$

(A) -4 (B) -2 (C) -1

(D) 0 (E) 2

3. $\lim_{\theta\to 0} \frac{1 - \cos\theta}{2\sin^2\theta}$ is

(A) 0 (B) $\frac{1}{8}$ (C) $\frac{1}{4}$

(D) 1 (E) nonexistent

4.If *f* is a differentiable function, then $f'(a)$ is given by which of the following?

I. $\lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$

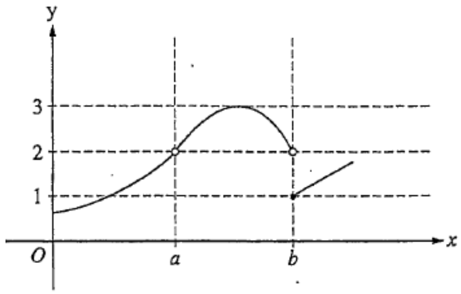
II. $\lim_{x\to a} \frac{f(x) - f(a)}{x - a}$

III. $\lim_{x\to a} \frac{f(x+h) - f(x)}{h}$

(A) I only (B) II only (C) I and II only

(D) I and III only (E) I, II, and III

5.



The graph of the function *f* is shown in the figure above. Which of the following statements about *f* is true?

(A) $\lim_{x \to a} f(x) = \lim_{x \to b} f(x)$

(B) $\lim_{x \to a} f(x) = 2$

(C) $\lim_{x \to b} f(x) = 2$

(D) $\lim_{x \to b} f(x) = 1$

(E) $\lim_{x \to a} f(x)$ does not exist.

6. Let f be the function defined by the following. For what values of x is f NOT continuous?

$$f(x) = \begin{cases} \sin x, & x < 0 \\ x^2, & 0 \le x < 1 \\ 2 - x, & 1 \le x < 2 \\ x - 3, & x \ge 2 \end{cases}$$

(A) 0 only
(B) 1 only
(C) 2 only
(D) 0 and 2 only
(E) 0, 1, and 2

7. If $f(x) = \begin{cases} \ln x & \text{for } 0 < x \le 2 \\ x^2 \ln 2 & \text{for } 2 < x \le 4, \end{cases}$ then $\lim_{x \to 2} f(x)$ is

(A) ln 2

(B) ln 8

(C) ln 16

(D) 4

(E) nonexistent

8. $\lim_{x \to 0} \dfrac{\tan \pi x}{x}$

(A) $\dfrac{1}{\pi}$ (B) 0 (C) 1 (D) π (E) ∞

x	0	1	2
$f(x)$	1	k	2

9.The function f is continuous on the closed interval [0, 2] and has values that are given in the table above. The equation $f(x)=\frac{1}{2}$ must have at least two solutions in the interval [0, 2] if k =

(A) 0 (B) $\frac{1}{2}$ (C) 1

(D) 2 (E) 3

*10. If $a \neq 0$, then $\lim_{x\to a} \frac{x^2-a^2}{x^4-a^4}$ is

(A) $\frac{1}{a^2}$ (B) $\frac{1}{2a^2}$ (C) $\frac{1}{6a^2}$

(D) 0 (E) nonexistent

11. Find g(x),that will make the function continuous at x =1.

$$f(x)=\begin{cases} 2x^2+3 & if\ x\geq 1 \\ g(x) & if\ x<1 \end{cases}$$

a. $\cos(x+4)$
b. x
c. 6-x
d. $2x^2-3$
e. x^2+2

12. $\lim_{h\to 0}\frac{(1+h)^6-1}{h}$

(A) 0 (B) 1 (C) 6 (D) ∞ (E) Does note exis

13. $\lim_{h\to 0}\frac{\sin(3x)}{\sin(4x)}$

(A) 1 (B) $\frac{4}{3}$ (C) $\frac{3}{4}$ (D) 0 (E) Does not exist

14. $\lim\limits_{x\to\infty} \dfrac{e^x}{x^{50}}$

(A) 0 (B) 1 (C) $\dfrac{1}{50!}$ (D) ∞ (E) none of these

15. $\lim\limits_{x\to-\infty} \dfrac{e^x}{x^{50}}$

(A) 0 (B) 1 (C) $\dfrac{1}{50!}$ (D) ∞ (E) none of these

16 . $\lim\limits_{h\to 0} \dfrac{\ln(e+h)-1}{h}$ is

(A) $f'(e)$, where $f(x)=\ln x$ (B) $f'(e)$, where $f(x)=\dfrac{\ln x}{x}$

(C) $f'(1)$, where $f(x)=\ln x$ (D) $f'(1)$, where $f(x)=\ln(x+e)$

(E) $f'(0)$, where $f(x)=\ln x$

FRQ:

Question 6

Let f be a function defined by $f(x) = \begin{cases} 1 - 2\sin x & \text{for } x \le 0 \\ e^{-4x} & \text{for } x > 0. \end{cases}$

(a) Show that f is continuous at $x = 0$.

(b) For $x \neq 0$, express $f'(x)$ as a piecewise-defined function. Find the value of x for which $f'(x) = -3$.

(c) Find the average value of f on the interval $[-1, 1]$.