AP Problem Packet – Mandatory HWK
2 daily grades – Cannot be dropped

name _____

1. A function $f(x)$ is continuous on on [-2,3] and has the properties for $f'(x)$ and $f''(x)$ given below.

| x | -2 | -2<x<0 | 0 | 0<x<1 | 1 | 1<x<3 | 3 |
|---|---|---|---|---|---|---|---|
| $f(x)$ | 0 | Positive | 2 | Positive | 3 | Positive | 1 |
| $f'(x)$ | DNE | Positive | 0 | Positive | DNE | Negative | DNE |
| $f''(x)$ | DNE | Negative | 0 | Positive | DNE | Negative | DNE |

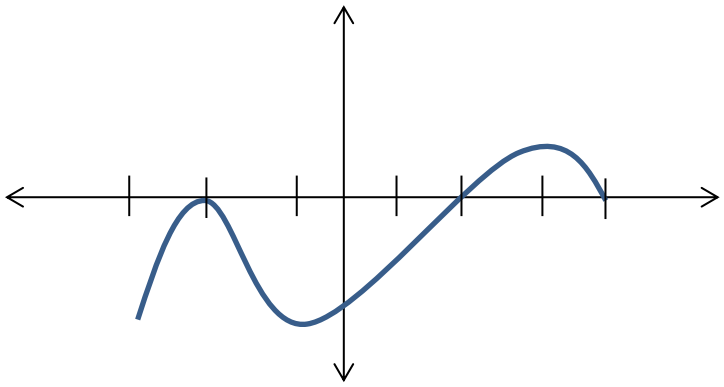
a) Find the x- values for any relative extrema. Identify if they are maximums or minimums and justify your conclusions.

b) Where is $f(x)$ concave up? Justify your answer.

c) Find any points of inflection. Justify your answer.

d) Sketch a graph of $f(x)$ on [-2,3] that satisfies the given information.

2. A function $f(x)$ is continuous on [-3,4] and the graph of $f'(x)$ is given below.
$f(-3) = 2,\ f(-1) = 0,\ and\ f(4) = 0$



a) What are the critical numbers for $f(x)$? Justify your conclusion.

b) Where does $f(x)$ have relative extrema? Is each extrema a relative maximum or a relative minimum? Justify your conclusion.

c) On what interval(s) is $f(x)$ concave down? Justify your conclusion.

d) Sketch a graph of $f(x)$ on [-3,4] that satisfies the given information.

3. A function $f(x)$ is continuous on on [-2,3] and has the properties for $f'(x)$ and $f''(x)$ given below.

| **x** | **-2** | **-2<x<0** | **0** | **0<x<1** | **1** | **1<x<3** | **3** |
|---|---|---|---|---|---|---|---|
| $\boldsymbol{f(x)}$ | 0 | Negative | -2 | Negative | 0 | Positive | 3 |
| $\boldsymbol{f'(x)}$ | DNE | Negative | DNE | Positive | 0 | Positive | DNE |
| $\boldsymbol{f''(x)}$ | DNE | Negative | DNE | Negative | 0 | Positive | DNE |

a) Find the x- values for any relative extrema. Identify if they are maximums or minimums and justify your conclusions.

b) Where is $f(x)$ increasing? Justify your answer.

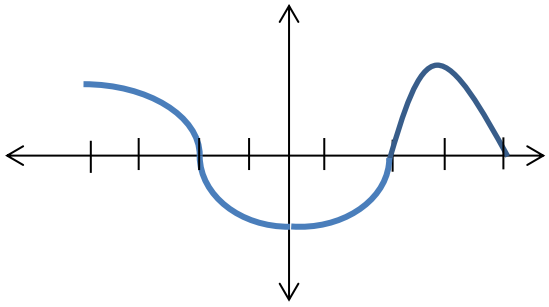
c) Find any points of inflection. Justify your answer.

d) Sketch a graph of $f(x)$ on [-2,3] that satisfies the given information.

4. A function $f(x)$ is continuous on on [-4,4] and the graph of $f'(x)$ is given below.

$f'(x)$ has a vertical tangent at x=-2 and horizontal tangents at x=0 and x=3.

$x = -4, 0, and\ 4\ are\ all\ roots\ of\ f(x).$



a) Where is $f(x)$ increasing? Justify your conclusion.

b) Where does $f(x)$ have relative extrema? Is each extrema a relative maximum or a relative minimum? Justify your conclusion.

c) Where is $f(x)$ concave down? Justify your conclusion.

d) Sketch a graph of $f(x)$ on [-4,4] that satisfies the given information assuming that $f(0) = 0$.

Multiple Choice Practice –

The answers are highlighted, you must justify the correct answer with proper AP justification.

1. If $f(x)=\sin\left(\frac{x}{2}\right)$, then there exists a number c in the interval $\frac{\pi}{2}<x<\frac{3\pi}{2}$ that satisfies the conclusion of the Mean Value Theorem. Which of the following could be c?

A) $\frac{2\pi}{3}$ B) $\frac{3\pi}{4}$ C) $\frac{5\pi}{6}$

D) $\pi$ E) $\frac{3\pi}{2}$

2. At what value of x does the graph of $y=\frac{1}{x^2}-\frac{1}{x^3}$ have a point of inflection?

A) 0 B) 1 C) 2

D) 3 E) At no value of x

3. The derivative of f is $x^4(x-2)(x+3)$. At how many points will the graph of f have a relative maximum?

A) none B) one C) two

D) three E) four

4. How many critical points does the function $f(x)=(x+2)^5(x-3)^4$ have?

A) one B) two C) three

D) five E) nine

5. Let f be the function with derivative given by $f'(x) = x^2 - \frac{5}{x}$, on which of the following intervals is f increasing.

A. $(-\infty, \infty)$

B. $(-\infty, 0) \cup (\sqrt[3]{5}, \infty)$

C. $(\sqrt[3]{5}, \infty)$ only

D. $(0, \sqrt[3]{5})$

6. Let f be the function defined by $f(x) = \begin{cases} x^3, & x \le 0 \\ x, & x > 0 \end{cases}$. Which of the following statements about f is true?

A) f is an odd function

B) f is discontinuous at x=0

C) f has a relative maximum

D) $f'(0) = 0$

E) $f'(x) > 0$ for $x \neq 0$

**Calculator questions**

7.** If the derivative of f is given by $f'(x) = e^x - 3x^2$ at which of the following values of x does f have a realtive maximum value?

A. -0.46

B. 0.20

C. 0.91

D. 0.95

E. 3.73

8.** The function f is given by $f(x) = x^3 + 12x - 24$ is

A) increasing for x<-2, decreasing for −2<x<2, increasing for x>2.
B) decreasing for x<0, increasing for x>0.
C) increasing for all x
D) decreasing for all x
E) decreasing for all x<-2, increasing for −2<x<2, decreasing for x>2.

9.** The function $f$ has a first derivative given by $f'(x) = \frac{x}{x^2 - x - 1}$. What is the x coordinate of the inflection point of the graph of $f$?

A. -0.618

B. 1.618

C. 0

D. -4.866

E. The graph of $f$ has no inflection point