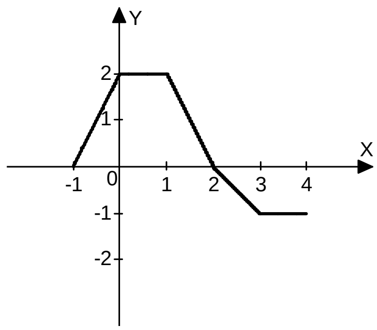
AP Practice – Integration name ________________

1. The graph of a piecewise linear function *f*, for $-1 \le x \le 4$, is shown below. What is the value of $\int_{-1}^{4} f(x)\,dx$?

(A) 1 (B) 2.5 (C) 4

(D) 5.5 (E) 8



2. $\int_{1}^{2} \frac{1}{x^2}\,dx =$

(A) $-\frac{1}{2}$ (B) $\frac{7}{24}$ (C) $\frac{1}{2}$

(D) 1 E) $2\ln 2$

3. $\int_{0}^{x} \sin\ t\ \ dt =$

(A) $\sin x$ (B) $-\cos x$ (C) $\cos x$

(D) $\cos x - 1$ (E) $1 - \cos x$

4. $\int_{1}^{e} \left(\frac{x^2-1}{x}\right) dx =$

(A) $e - \frac{1}{e}$ (B) $e^2 - e$ (C) $\frac{e^2}{2} - e + \frac{1}{2}$

(D) $e^2 - 2$ (E) $\frac{e^2}{2} - \frac{3}{2}$

5. If f is a linear function and $0 < a < b$, then $\int_{a}^{b} f''(x)\,dx =$

(A) 0 (B) 1 (C) $\frac{ab}{2}$

(D) $b - a$ (E) $\frac{b^2 - a^2}{2}$

6. If $F(x)=\int_0^x \sqrt{t^3+1}\,dt$, then $F'(2)=$

(A) -3 (B) -2 (C) 2

(D) 3 (E) 18

7. What are all values of k for which $\int_{-3}^{k} x^2\,dx = 0$?

(A) -3 (B) 0 (C) 3

(D) -3 and 3 (E) -3, 0, and 3

8. If $\frac{dy}{dt}=ky$ and k is a nonzero constant, then y could be

(A) $2e^{kty}$ (B) $2e^{kt}$ (C) $e^{kt}+3$

(D) $kty+5$ (E) $\frac{1}{2}ky^2+\frac{1}{2}$

9. What is the average value of $y=x^2\sqrt{x^3+1}$ on the interval [0, 2]?

(A) $\frac{26}{9}$ (B) $\frac{52}{9}$ (C) $\frac{26}{3}$

(D) $\frac{52}{3}$ (E) 24

*10. If f is a continuous function and if $F'(x)=f(x)$ for all real numbers x, then $\int_1^3 f(2x)\,dx=$

(A) $2F(3)-2F(1)$ (B) $\frac{1}{2}F(3)-\frac{1}{2}F(1)$ (C) $2F(6)-2F(2)$

(D) $F(6)-F(2)$ (E) $\frac{1}{2}F(6)-\frac{1}{2}F(2)$

x	2	5	7	8
$f(x)$	10	30	40	20

*11. The function f is continuous on the closed interval $[2, 8]$ and has values that are given in the table above. Using the subintervals $[2, 5]$, $[5, 7]$, $[7, 8]$, what is the trapezoidal approximation of $\int_2^8 f(x)\,dx$

(A) 110 (B) 130 (C) 160

(D) 190 (E) 210

*12. Let $F(x)$ be an antiderivative of $\frac{(\ln x)^3}{x}$. If $F(1) = 0$, then $F(9) =$

(A) 0.048 (B) 0.144 (C) 5.827

(D) 23.308 (E) 1,640.250

13. $\int_0^1 e^{-4x}\,dx =$

(A) $\frac{-e^{-4}}{4}$ (B) $-4e^{-4}$ (C) $e^{-4}-1$

(D) $\frac{1}{4}-\frac{e^{-4}}{4}$ (E) $4-4e^{-4}$

14. $\int_0^{\frac{\pi}{4}} \sin x\ dx =$

(A) $-\frac{\sqrt{2}}{2}$ (B) $\frac{\sqrt{2}}{2}$ (C) $-\frac{\sqrt{2}}{2}-1$

(D) $-\frac{\sqrt{2}}{2}+1$ (E) $\frac{\sqrt{2}}{2}-1$

15. $\int x^2\cos\left(x^3\right)dx =$

(A) $-\frac{1}{3}\sin\left(x^3\right)+C$ (B) $\frac{1}{3}\sin\left(x^3\right)+C$ (C) $-\frac{x^3}{3}\sin\left(x^3\right)+C$

(D) $\frac{x^3}{3}\sin\left(x^3\right)+C$ (E) $\frac{x^3}{3}\sin\left(\frac{x^4}{4}\right)+C$

16. Using the substitution $u = 2x + 1$, $\int_0^2 \sqrt{2x+1}\,dx$ is equivalent to

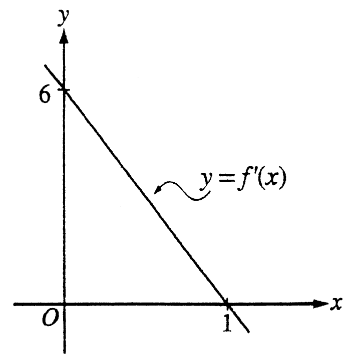
(A) $\frac{1}{2}\int_{-1/2}^{1/2} \sqrt{u}\,du$ (B) $\frac{1}{2}\int_0^2 \sqrt{u}\,du$ (C) $\frac{1}{2}\int_1^5 \sqrt{u}\,du$

(D) $\int_0^2 \sqrt{u}\,du$ (E) $\int_1^5 \sqrt{u}\,du$

17. The graph of f', the derivative of f, is the line shown in the figure above. If $f(0) = 5$, then $f(1) =$

(A) 0 (B) 3 (C) 6

(D) 8 (E) 11



18. $\frac{d}{dx}\left(\int_0^{x^2} \sin(t^3)\,dt\right) =$

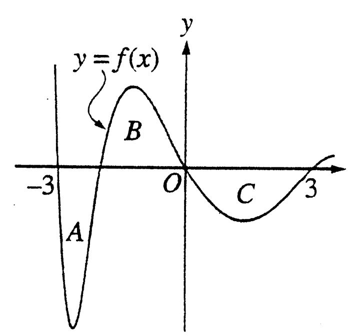
(A) $-\cos(x^6)$ (B) $\sin(x^3)$ C) $\sin(x^6)$

(D) $2x\sin(x^3)$ (E) $2x\sin(x^6)$

19. The regions A, B and C in the figure above are bounded by the graph of the function f and the x-axis. If the area of each region is 2, what is the value of $\int_{-3}^{3}(f(x)+1)\,dx$?

(A) –2 (B) –1 (C) 4

(D) 7 (E) 12



*20. The rate of change of the altitude of a hot-air balloon is given by $r(t) = t^3 - 4t^2 + 6$ for $0 \le t \le 8$. Which of the following expressions gives the change in altitude of the balloon during the time the altitude is decreasing?

(A) $\int_{1.572}^{3.514} r(t)\,dt$ (B) $\int_0^8 r(t)\,dt$ (C) $\int_0^{2.667} r(t)\,dt$

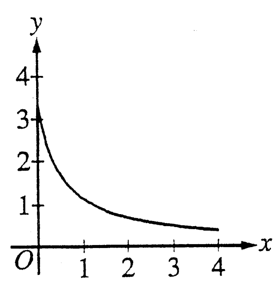
(D) $\int_{1.572}^{3.514} r'(t)\,dt$ (E) $\int_0^{2.667} r'(t)\,dt$

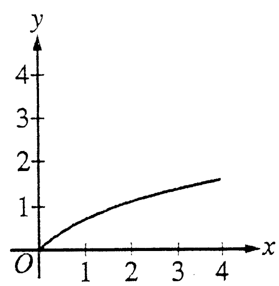
*21. The velocity, in ft/sec, of a particle moving along the *x*-axis is given by the function $v(t) = e^t + te^t$. What is the average velocity of the particle from time $t = 0$ to time $t = 3$?

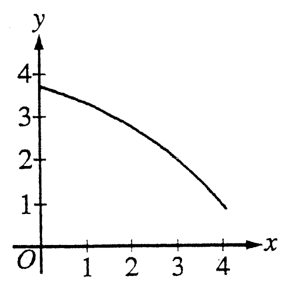
(A) 20.086 ft/sec (B) 26.447 ft/sec (C) 32.809 ft/sec

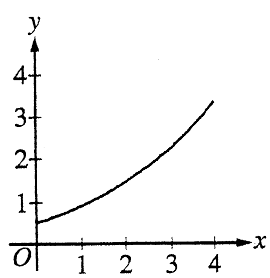
(D) 40.671 ft/sec (E) 79.342 ft/sec

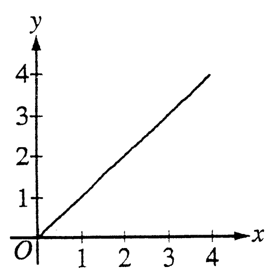
*22. If a trapezoidal sum overapproximates $\int_0^4 f(x)\,dx$, and a right Riemann sum underapproximates $\int_0^4 f(x)\,dx$, which of the following could be the graph of $y = f(x)$?

(A)


(B)


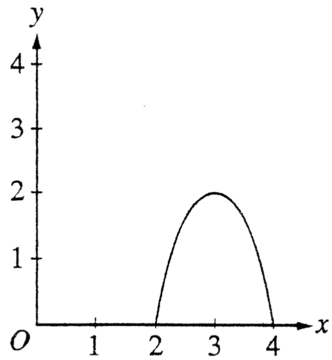
(C)


(D)


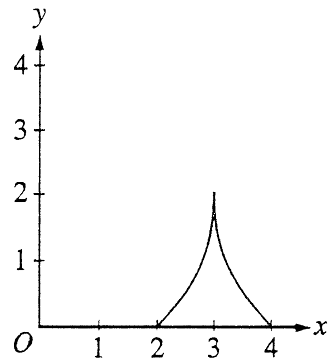
(E)


*23. On the closed interval $[2,4]$, which of the following could be the graph of a function f with the property that $\frac{1}{4-2}\int_2^4 f(t)\,dt = 1$?

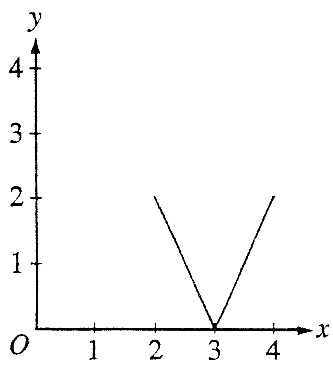
(A)



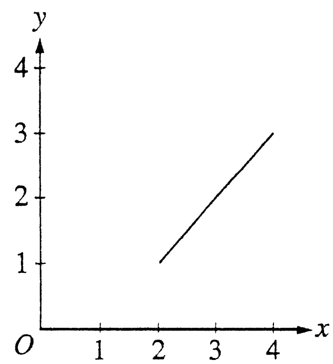
(B)



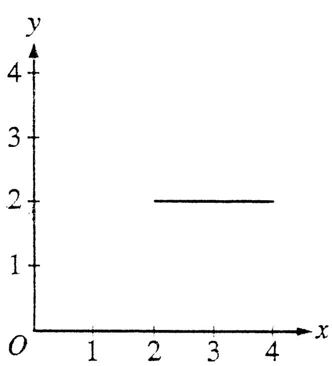
(C)



(D)



(E)



t (hours)	0	2	5	7	8
$E(t)$ (hundreds of entries)	0	4	13	21	23

2. A zoo sponsored a one-day contest to name a new baby elephant. Zoo visitors deposited entries in a special box between noon $(t = 0)$ and 8 P.M. $(t = 8)$. The number of entries in the box t hours after noon is modeled by a differentiable function E for $0 \le t \le 8$. Values of $E(t)$, in hundreds of entries, at various times t are shown in the table above.

(a) Use the data in the table to approximate the rate, in hundreds of entries per hour, at which entries were being deposited at time $t = 6$. Show the computations that lead to your answer.

(b) Use a trapezoidal sum with the four subintervals given by the table to approximate the value of $\frac{1}{8}\int_0^8 E(t)\,dt$. Using correct units, explain the meaning of $\frac{1}{8}\int_0^8 E(t)\,dt$ in terms of the number of entries.

(c) At 8 P.M., volunteers began to process the entries. They processed the entries at a rate modeled by the function P, where $P(t) = t^3 - 30t^2 + 298t - 976$ hundreds of entries per hour for $8 \le t \le 12$. According to the model, how many entries had not yet been processed by midnight $(t = 12)$?

(d) According to the model from part (c), at what time were the entries being processed most quickly? Justify your answer.