AP Motion Problems – Only use a calculator on  problems.

t (sec)	0	15	25	30	35	50	60
v(t) (ft/sec)	-20	-30	-20	-14	-10	0	10
a(t) (ft/sec^2	1	5	2	1	2	4	2

1. A car travels on a straight track. During the time interval $0 \leq t \leq 60$ seconds, the car's velocity *v*, measured in feet per second, and *a*, measured in feet per second per second, are continuous functions. The table above shows selected values of these functions.

(a) Using appropriate units, explain the meaning of $\int_{30}^{60} |v(t)|dt$ in terms of the car's motion. Approximate $\int_{30}^{60} |v(t)|dt$ using a trapezoidal approximation with the three subintervals determined by the table.

(b) Using appropriate units, explain the meaning of $\int_{0}^{30} a(t)dt$ in terms of the car's motion. Find the exact value of $\int_{0}^{30} a(t)dt$.

(c) For $0 \leq t \leq 60$, must there be a time t when *v(t)* =-5? Justify your answer.

(d) For $0 \leq t \leq 60$, must there be a time when *a(t)* = 0? Justify your answer.

t (seconds)	0	10	20	30	40	50	60	70	80
v(t) (feet per second)	5	14	22	29	35	40	44	47	49

2. Rocket A has positive velocity v(t) after being launched upward form an initial height 0 feet at time t=0 seconds. The velocity of the rocket is recorded for selected values of t over the interval of $0 \leq t \leq 80$ seconds, as shown in the table above.

   (a)Find the average acceleration of rocket A over the time interval $0 \leq t \leq 80$ seconds. Indicate units of measure.

   (b)Using correct units, explain the meaning of $\int_{10}^{70} v(t)dt$ in terms of the rocket's flight. Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\int_{10}^{70} v(t)dt$.

   (c) Rocket B is launched upward with an acceleration of $a(t) = \frac{3}{\sqrt{t+1}}$ feet per second per second. At time t=0 seconds, the initial height of the rocket is 0 feet, and the initial velocity is 2 feet per second. Which of the two rockets is travelling faster at time t=80 seconds? Explain your answer.

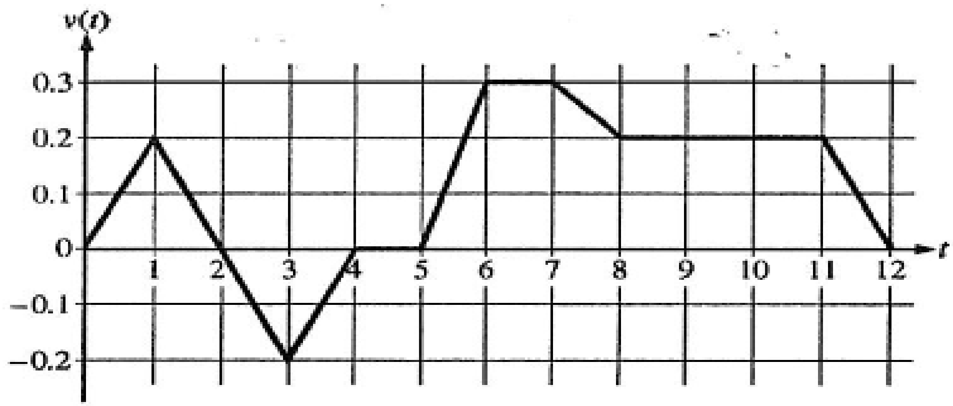
3. A particle moves along the y axis so that its velocity v at time $t \geq 0$ is given by $v(t) = 1 - \tan^{-1}(e^t)$. At time t=0, the particle is at y=-1. (Note $\tan^{-1}x = \arctan x$)

   (a) Find the acceleration of the particle at time t=2.

   (b) Is the speed of the particle increasing or decreasing at time t=2? Give a reason for your answer.

   (c) Find the time $t \geq 0$ at which the particle reaches its highest point. Justify your answer.

   (d) Find the position of the particle at time t=2. Is the particle moving toward the origin or away from the origin at time t=2? Justify your answer



4. Caren rides her bicycle along a straight road from home to school, starting at home at time t=0 minutes and arriving at school at time t=12 minutes. During the time interval $0 \leq t \leq 12$ minutes her velocity v(t), in miles per minute is modeled by the piecewise-linear function who's graph is shown above.

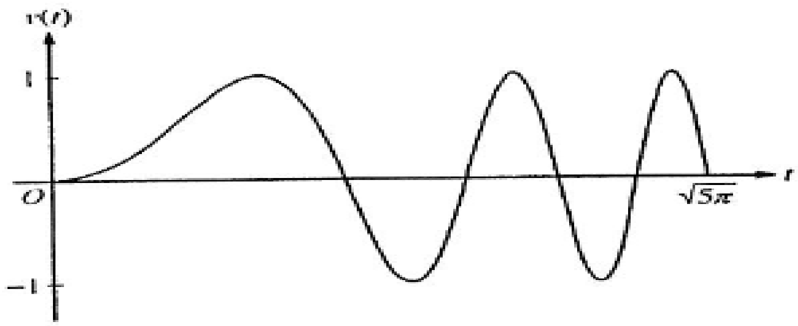
   (a) Find the acceleration of Caren's bicycle at time t=7.5 minutes. Indicate units of measure.

   (b) Using correct units, explain the meaning of $\int_0^{12} |v(t)|dt$ in terms of Caren's trip. Find the value of $\int_0^{12} |v(t)|dt$.

   (c) Shortly after leaving home, Caren realizes she left her calculus homework at home, and she returns to get it. At what time does she turn around to go back home? Give a reason for your answer.

   (d) Larry also rides his bicycle along a straight road from home to school in 12 minutes. His velocity is modeled by the function w given by $w(t) = \frac{\pi}{15}\sin(\frac{\pi}{12}t)$, where w(t) is in miles per minute for $0 \leq t \leq 12$ minutes.

   Who lives closer to school: Caren or Larry? Show the work that leads to your answer.



5. A particle moves along the x-axis so that it's velocity v at time $t \geq 0$ is given by v(t)=sin(t²). The graph of v is shown above for $0 \leq t \leq \sqrt{5\pi}$. The position of the particle at time t is x(t) and its position at time t=0 is x(0)=5.

   (a) Find the acceleration of the particle at time t=3.

   (b) Find the total distance traveled by the particle from time t=0 to t=3.

   (c) Find the position of the particle at time t=3.

   (d) For $0 \leq t \leq \sqrt{5\pi}$, find the time t at which the particle is farthest to the right. Explain your answer.

6. A particle moves along the x-axis with velocity at time t≥ 0 given by $-1 + e^{1-t}$.
   (a) Find the acceleration of the particle at time t=3.

   (b) Is the speed of the particle increasing t=3? Give a reason for your answer.

   (c) Find all values of t at which the particle changes direction. Justify your answer.

   (d) Find the total distance traveled by the particle over the time $0 \leq t \leq 3$.

7. A particle moves along the x-axis so that its velocity at time t is given by:

$$v(t) = -(t+1)\sin(\frac{t^2}{2}).$$

At time t=0, the particle is at position x=1.

(a) Find the acceleration of the particle at time t=2. Is the speed of the particle increasing at t=2? Why or why not?

(b) Find all times t in the open interval $0<t<3$ when the particle changes direction. Justify your answer.

(c) Find the total distance traveled by the particle from time t=0 until time t=3.

(d) During the time interval $0 \le t \le 3$, what is the greatest distance between the particle and the origin. Show the work that leads to your answer.

8. For $0 \le t \le 6$, a particle is moving along the x-axis. The particle's position x(t), is not explicitly given. The velocity of the particle is given by $v(t) = 2\sin\left(e^{\frac{t}{4}}\right) + 1$. The acceleration of the particle is given by $a(t) = \frac{1}{2}e^{\frac{t}{4}}\cos\left(e^{\frac{t}{4}}\right)$ and x(0)=2.

(a) Is the speed of the particle increasing or decreasing at time t=5.5? Give a reason for your answer.

(b) Find the average velocity of the particle for the time period $0 \le t \le 6$.

(c) Find the total distance traveled by the particle from time t=0 to t=6.

(d) For $0 \le t \le 6$, the particle changes direction exactly once. Find the position of the particle at that time.