AP MOTION KEY!

1. a) $\int_{30}^{60} |v(t)|\,dt$ is the total distance in ft that the car travels during the time $t = 30$ secs to $t = 60$ sec.

$$\underset{30 \le t \le 60}{\text{TOTAL DISTANCE}} \approx \left(\frac{14+10}{2}\cdot 5 + \frac{10+0}{2}\cdot 15 + \frac{0+10}{2}\cdot 10\right) ft.$$

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b) $\int_{0}^{30} a(t)\,dt$ is the change in velocity in ft/sec of the car during the first 30 seconds.

$$\int_{0}^{30} a(t)\,dt = \int_{0}^{30} v'(t)\,dt = v(30) - v(0) = [-14 - (-20)]\ ft/sec$$

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c) IVT - yes

$v(35) < -5 < v(50)$

$v(t)$ is continuous on $35 \le t \le 50$

$\therefore$ $v(t)$ must $= -5$ somewhere on this interval.

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d) $a(t) = v'(t)$

$v'(t) = 0$ on $0 \le t \le 60$ because

$v(t)$ is cont. on $0 \le t \le 60$ and diff. on $0 < t < 60$

and $v(0) = -20$ and $v(25) = -20$

$\therefore$ $v'(t) = 0$ somewhere on $0 < t < 25$ By Rolle's Thm.

2. a) Avg accel $_{0 \le t \le 80}$ $= \dfrac{1}{80}\displaystyle\int_0^{80} a(t)\,dt$

$$= \frac{1}{80}\int_0^{80} v'(t)\,dt$$

$$= \frac{1}{80}\left[v(80) - v(0)\right] = \frac{49-5}{80}\ ft/sec^2$$

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b) $\displaystyle\int_{10}^{70} v(t)\,dt$ is the Rocket's change in height in ft from 10 seconds to 70 seconds.

$\approx [(22 \cdot 20) + (35 \cdot 20) + (44 \cdot 20)]$ ft

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c) Rocket B:

$x_B(0) = 0$

$v_B(0) = 2$

$$2 + \int_0^{80} a_B(t)\,dt = v(80)$$

$$2 + \left[6\sqrt{t+1}\Big|_0^{80}\right]$$

$$2 + [6 \cdot 9 - 6] = 50\ ft/sec$$

ROCKET A: $v(80) = 49$

Rocket B is traveling faster when $t = 80$.

3. a) $v'(t) = a(t)$

$a(2) = v'(2) = -.132$ or $-.133$

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b) speed $= |v(t)|$

$v(2) < 0 \qquad a(t) < 0$

since $v(2)$ & $a(2)$ have the same sign, speed is increasing when $t = 2$.

OR

the slope of $|v(t)|$ at $t = 2$ is $+$, $\therefore$

speed is increasing when $t = 2$

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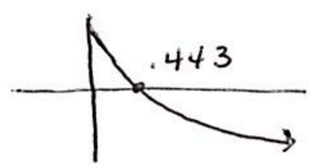
c) The highest point is the absolute maximum.

t	$y(t)$
0	-1
$.443$	$\int_0^{.443} v(t)\,dt = -.953$



$v(t)$ changes from $+$ to $-$ @ $t = .443$

Because $t = .443$ is the only rel. max, it must be the absolute max.

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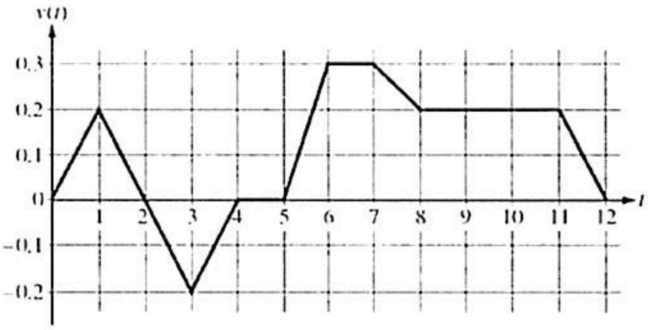
d) $y(2) = -1 + \int_0^2 v(t)\,dt = -1.360$ or -1.361

$v(2) = -.436 \;\therefore$ the particle is moving down, and away from the origin when $t = 2$.

AP® CALCULUS AB
2009 SCORING GUIDELINES

Question 1



Caren rides her bicycle along a straight road from home to school, starting at home at time $t = 0$ minutes and arriving at school at time $t = 12$ minutes. During the time interval $0 \le t \le 12$ minutes, her velocity $v(t)$, in miles per minute, is modeled by the piecewise-linear function whose graph is shown above.

(a) Find the acceleration of Caren's bicycle at time $t = 7.5$ minutes. Indicate units of measure.

(b) Using correct units, explain the meaning of $\int_0^{12} |v(t)|\, dt$ in terms of Caren's trip. Find the value of $\int_0^{12} |v(t)|\, dt$.

(c) Shortly after leaving home, Caren realizes she left her calculus homework at home, and she returns to get it. At what time does she turn around to go back home? Give a reason for your answer.

(d) Larry also rides his bicycle along a straight road from home to school in 12 minutes. His velocity is modeled by the function w given by $w(t) = \frac{\pi}{15}\sin\left(\frac{\pi}{12}t\right)$, where $w(t)$ is in miles per minute for $0 \le t \le 12$ minutes. Who lives closer to school: Caren or Larry? Show the work that leads to your answer.

(a) $a(7.5) = v'(7.5) = \dfrac{v(8) - v(7)}{8 - 7} = -0.1$ miles/minute^2	$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{units} \end{cases}$
(b) $\int_0^{12} \lvert v(t)\rvert\, dt$ is the total distance, in miles, that Caren rode during the 12 minutes from $t = 0$ to $t = 12$. $\int_0^{12} \lvert v(t)\rvert\, dt = \int_0^{2} v(t)\, dt - \int_2^{4} v(t)\, dt + \int_4^{12} v(t)\, dt$ $= 0.2 + 0.2 + 1.4 = 1.8$ miles	$2 : \begin{cases} 1 : \text{meaning of integral} \\ 1 : \text{value of integral} \end{cases}$
(c) Caren turns around to go back home at time $t = 2$ minutes. This is the time at which her velocity changes from positive to negative.	$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$
(d) $\int_0^{12} w(t)\, dt = 1.6$; Larry lives 1.6 miles from school. $\int_0^{12} v(t)\, dt = 1.4$; Caren lives 1.4 miles from school. Therefore, Caren lives closer to school.	$3 : \begin{cases} 2 : \text{Larry's distance from school} \\ \quad 1 : \text{integral} \\ \quad 1 : \text{value} \\ 1 : \text{Caren's distance from school} \\ \quad \text{and conclusion} \end{cases}$

* 5. a) $a(t) = v'(t)$

$a(3) = -5.466$ or -5.467

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b) TOTAL DISTANCE $0 \le t \le 3$ $= \int_0^3 |v(t)|\,dt = 1.702$

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c) $x(3) = 5 + \int_0^3 v(t)\,dt = 5.773$ or 5.774

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d) Farthest to the right = Absolute Maximum

t	$x(t)$
0	5
1.772	5.895
2.506	Rel min
3.069	5.788
3.544	Rel min
$\sqrt{5\pi}$	5.752

$v(t) = 0$

when $t = 1.772, 2.506, 3.069, 3.544$

The particle is farthest to the right when $t = 1.772$.

$x(a) = 5 + \int_0^a v(t)\,dt$

6. a) $a(t) = v'(t)$

$v'(t) = -e^{1-t}$

$v'(3) = -e^{-2}$

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b) speed $= |v(t)|$

$v(3) = -1 + e^{-2} < 0$

$a(3) = -e^{-2} < 0$

$\therefore$ the speed of the particle is increasing when $t = 3$.

OR

the slope of $|v(3)|$ is positive. $\therefore$ the speed is increasing when $t = 3$.

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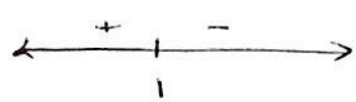
c) The particle changes directions when $v(t)$ changes signs.



$v(t) = 0$ when

$t = 1$

$-1 + e^{1-t} = 0$

$e^{1-t} = 1$

$\ln 1 = 1 - t$

$t = 1$

The particle will change directions at $t = 1$ because $v(t)$ changes signs at this t-value.

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d) TOTAL DISTANCE $0 \le t \le 3$ $= \int_0^3 |v(t)|\,dt$

$$= \int_0^1 v(t)\,dt + \int_1^3 -v(t)\,dt = -t - e^{1-t}\Big|_0^1 - (-t - e^{1-t})\Big|_1^3$$

$$= e + e^{-2} - 1$$

* 7. a) $a(t) = v'(t)$

$a(2) = v'(2) = 1.587$ or 1.588

$a(2) > 0$ $\therefore$ speed is decreasing

$v(2) < 0$ when $t = 2$.

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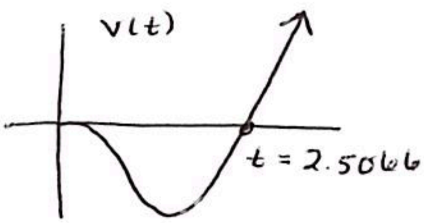
b) The particle changes directions when $v(t)$ changes signs.



velocity changes from $-$ to $+$ when $t = 2.5066$.

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c) TOTAL DISTANCE $0 \le t \le 3$ $\int_0^3 |v(t)|\,dt = 4.333$ or 4.334

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d)

t	$x(t)$
0	1
2.5066	$1 + \int_0^{2.5066} v(t)\,dt = -2.265$
3	$1 + \int_0^3 v(t)\,dt = -1.197$

Absolute max or min

critical values when $v(t) = 0$ $\therefore$ @ $t = 2.506$

The particle is furthest from the origin when $t = 2.506$. The distance is 2.265.

(2011) 1. $v(t) = 2\sin e^{t/4} + 1$

$a(t) = \frac{1}{2} e^{t/4} \cos e^{t/4}$

$x(0) = 2$

a) speed is increasing if $a(t)$ and $v(t)$ have the same sign

$v(5.5) = -0.453 < 0$ so speed is

$a(5.5) = -1.3585 < 0$ increasing @ $t = 5.5$

b) average velocity $= \dfrac{\int_0^6 v(t)\,dt}{6} = 1.949$

c) total distance $= \int_0^6 |v(t)|\,dt = 12.573$

d) particle changes direction when $v(t)$ changes sign

$v(t) = 0$ when $t = 5.195$ and changes from (+) to (−) at this t value

$2 + \int_0^{5.195} v(t)\,dt = 14.134$