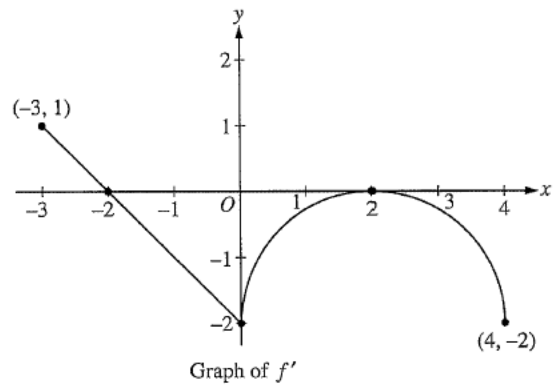
Derivative Applications
AP Practice

2003 AB 4 and BC 4



Graph of f'

Let f be a function defined on the closed interval $-3 \le x \le 4$ with $f(0) = 3$. The graph of f', the derivative of f, consists of one line segment and a semicircle, as shown above.

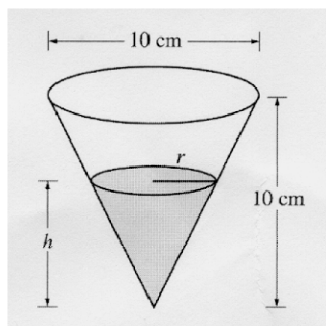
(a) On what intervals, if any, is f increasing? Justify your answer.

(b) Find the x-coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$. Justify your answer.

(c) Find an equation for the line tangent to the graph of f at the point (0,3).

2002 AB 5

A container has the shape of an open right circular cone, as shown in the figure below. The height of the container is 10 cm and the diameter of the opening is 10 cm. Water in the container is evaporating so that its depth h is changing at the constant rate of $-\frac{3}{10}$ cm/hr.



(Note: the volume of a cone of height h and radius r is given by $V = \frac{1}{3}\pi r^2 h$.)

(a) Find the volume V in the container when $h = 5$ cm. Indicate units of measure.

(b) Find the rate of change of the volume of water in the container, with respect to time, when $h = 5$ cm. Indicate units of measure.

(c) Show that the rate of change of the volume of water in the container due to evaporation is directly proportional to the exposed surface area of the water. What is the constant of proportionality?

1984 AB 4 and BC 3

1984 AB 4 and BC 3

1984 AB 4 and BC 3

A function *f* is continuous on the closed interval $[-3,3]$ such that $f(-3)=4$ and $f(3)=1$. The function f' and f'' have the properties given in the table below.

x	$-3<x<-1$	$x=-1$	$-1<x<1$	$x=1$	$1<x<3$
$f'(x)$	Positive	Fails to exist	Negative	0	Negative
$f''(x)$	Positive	Fails to exits	Positive	0	Negative

(a) What are the *x*-coordinates of all absolute maximum and absolute minimum points of *f* on the interval $[-3,3]$? Justify your answer.

(b) What are the *x*-coordinates of all points of inflection of *f* on the interval $[-3,3]$? Justify your answer.

(c) Sketch a graph that satisfies the given properties of *f*.

1.What is the *x*-coordinate of the point of inflection on the graph of $y=\frac{1}{3}x^3+5x^2+24$?

(A) 5 (B) 0 (C) $-\frac{10}{3}$

(D) – 5 (E) – 10

2.A particle moves along the *x*-axis so that its position at time *t* is given by $x(t)=t^2-6t+5$. For what value of t is the velocity of the particle zero?

(A) 1 (B) 2 (C) 3

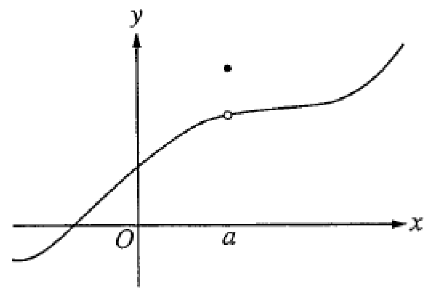
(D) 4 (E) 5

3.The maximum acceleration attained on the interval $0 \le t \le 3$ by the particle whose velocity is given by $v(t)=t^3-3t^2+12t+4$ is

(A) 9 (B) 12 (C) 14

(D) 21 (E) 40

* 4. The graph of a function f is shown below. Which of the following statements about f is false?



(A) f is continuous at $x = a$.

(B) f has a relative maximum at $x = a$.

(C) $x = a$ is in the domain of f.

(D) $\lim_{x \to a^+} f(x)$ is equal to $\lim_{x \to a^-} f(x)$.

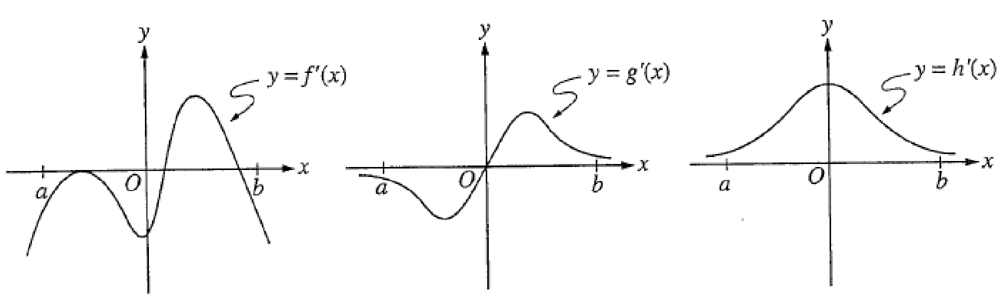
(E) $\lim_{x \to a} f(x)$ exists

* 5. The radius of a circle is decreasing at a constant rate of 0.1 centimeter per second. In terms of the circumference C, what is the rate of change of the area of the circle, in square centimeters per second?

(A) $-(0.2)\pi C$ (B) $-(0.1)C$ (C) $-\frac{(0.1)C}{2\pi}$

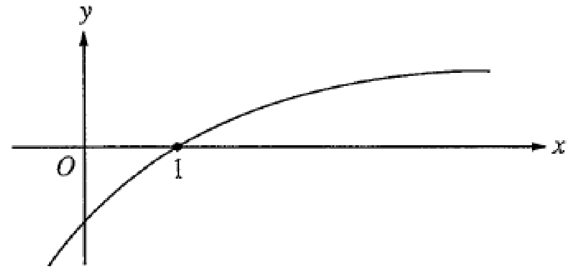
(D) $(0.1)^2 C$ (E) $(0.1)^2 \pi C$

* 6. The graphs of the derivatives of the functions f, g, and h are shown below. Which of the functions f, g, or h have a relative maximum on the open interval $a < x < b$?



(A) f only (B) g only (C) h only

(D) f and g only (E) f, g, and h



7. The graph of a twice-differentiable function f is shown in the figure above. Which of the following is true?

(A) $f(1) < f'(1) < f''(1)$ (B) $f(1) < f''(1) < f'(1)$

(C) $f'(1) < f(1) < f''(1)$ (D) $f''(1) < f(1) < f'(1)$ (E) $f''(1) < f'(1) < f(1)$

8. If $f''(x) = x(x+1)(x-2)^2$, then the graph of f has inflection points when $x =$

(A) –1 only (B) 2 only (C) – 1 and 0 only

(D) – 1 and 2 only (E) – 1, 0, and 2 only

9. The function *f* is given by $f(x) = x^4 + x^2 - 2$. On which of the following intervals is *f* increasing?

(A) $\left(-\frac{1}{\sqrt{2}}, \infty\right)$ (B) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ (C) $(0, \infty)$

(D) $(-\infty, 0)$ (E) $\left(-\infty, -\frac{1}{\sqrt{2}}\right)$

10. The first derivative of the function f is given by $f'(x) = \frac{\cos^2 x}{x} - \frac{1}{5}$. How many critical values does f have on the open interval (0, 10)?

(A) One (B) Three (C) Four

(D) Five (E) Seven

11. If g is a differentiable function such that $g(x) < 0$ for all real numbers x and if $f'(x) = (x^2 - 4)g(x)$, which of the following is true?

(A) f has a relative maximum at $x = -2$ and a relative minimum at $x = 2$.

(B) f has a relative minimum at $x = -2$ and a relative maximum at $x = 2$.

(C) f has relative minima at $x = -2$ and at $x = 2$.

(D) f has relative maxima at $x = -2$ and at $x = 2$.

(E) It cannot be determined if f has any relative extrema.

12. The graph of f', the derivative of the function f, is shown above. Which of the following statements is true about f?

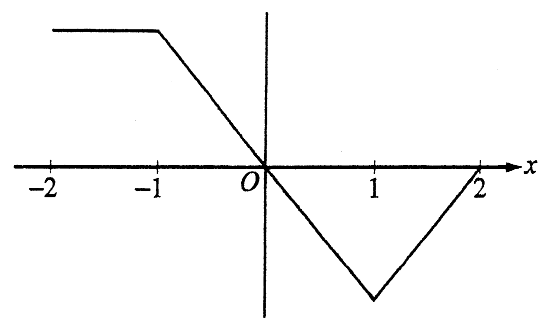
(A) f is decreasing for $-1 \le x \le 1$.

(B) f is increasing for $-2 \le x \le 0$.

(C) f is increasing for $1 \le x \le 2$.

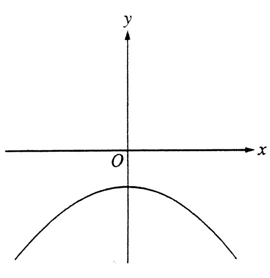
(D) f has a local minimum at $x = 0$.

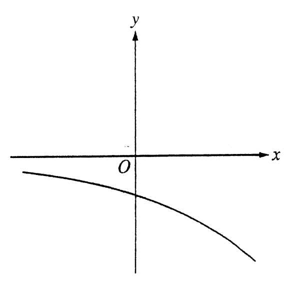
(E) f is not differentiable at $x = -1$ and $x = 1$.



Graph of f'

13. The function f has the property that $f(x)$, $f'(x)$, and $f''(x)$ are negative for all real values x. Which of the following could be the graph of f?

(A)


(B)


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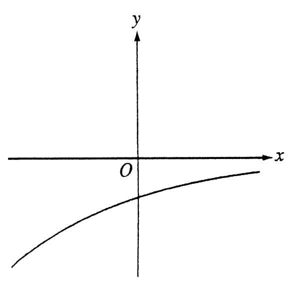
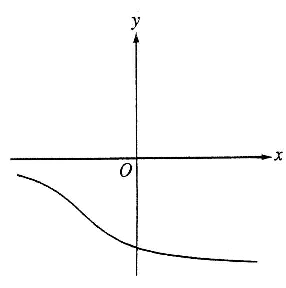
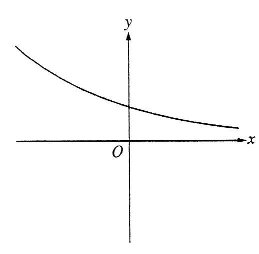


(D)


(E)


14. Let f be the function with derivative given by $f'(x) = x^2 - \frac{2}{x}$. On which of the following intervals is f decreasing?

(A) $(-\infty, -1]$ only (B) $(-\infty, 0)$ (C) $[-1, 0)$ only

(D) $(0, \sqrt[3]{2}]$ (E) $[\sqrt[3]{2}, \infty)$

15. Let f be the function given by $f(x) = 2xe^x$. The graph of f is concave down when

(A) $x < -2$ (B) $x > -2$ (C) $x < -1$

(D) $x > -1$ (E) $x < 0$

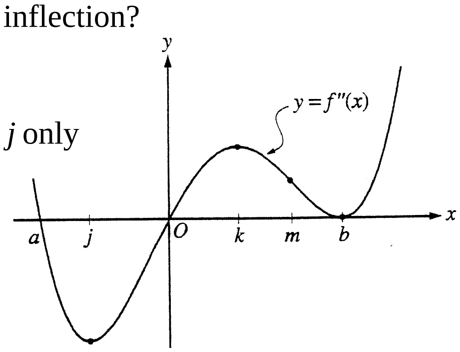
x	–4	–3	–2	–1	0	1	2	3	4
$g'(x)$	2	3	0	–3	–2	–1	0	3	2

16. The derivative g' of a function g is continuous and has exactly two zeros. Selected values of g' are given in the table above. If the domain of g is the set of all real numbers, then g is decreasing on which of the following intervals?

(A) $-2 \le x \le 2$ only (B) $-1 \le x \le 1$ only (C) $x \ge -2$

(D) $x \ge 2$ only (E) $x \le -2$ or $x \ge 2$

17. The second derivative of the function f is given by $f''(x) = x(x-a)(x-b)^2$. The graph of f'' is shown below. For what values of x does the graph of f have a point of inflection?



(A) 0 and a only (B) 0 and m only (C) b and j only

(D) 0, a, and b (E) b, j and k

*18. Let f be the function with derivative given by $f'(x) = \sin(x^2 + 1)$. How many relative extrema does f have on the interval $2 < x < 4$?

(A) One (B) Two (C) Three

(D) Four (E) Five

*19 For all x in the closed interval $[2,5]$, the function f has a positive first derivative and a negative second derivative. Which of the following could be a table of values for f?

(A)

x	$f(x)$
2	7
3	9
4	12
5	16

(B)

x	$f(x)$
2	7
3	11
4	14
5	16

(C)

x	$f(x)$
2	16
3	12
4	9
5	7

(D)

x	$f(x)$
2	16
3	14
4	11
5	7

(E)

x	$f(x)$
2	16
3	13
4	10
5	7

*20. A railroad track and a road cross at right angles. An observer stands on the road 70 meters south of the crossing and watches an eastbound train traveling at 60 meters per second. At how many meters per second is the train moving away from the observer 4 seconds after it passes through the intersection?

(A) 57.60 (B) 57.88 (C) 59.20

(D) 60.00 (E) 67.40

21. The top of a 25-foot ladder is sliding down a vertical wall at a constant rate of 3 feet per minute. When the top of the ladder is 7 feet from the ground, what is the rate of change of the distance between the bottom of the ladder and the wall?

(A) $-\frac{7}{8}$ feet per minute (B) $-\frac{7}{24}$ feet per minute

(C) $\frac{7}{24}$ feet per minute (D) $\frac{7}{8}$ feet per minute (E) $\frac{21}{25}$ feet per minute