AP Practice Packet

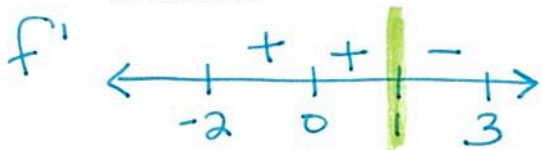
Name: Key

1. A function $f(x)$ is continuous on on [-2,3] and has the properties for $f'(x)$ and $f''(x)$ given below.

x	-2	-2<x<0	0	0<x<1	1	1<x<3	3
$f(x)$	0	Positive	2	Positive	3	Positive	1
$f'(x)$	DNE	Positive	0	Positive	DNE	Negative	DNE
$f''(x)$	DNE	Negative	0	Positive	DNE	Negative	DNE

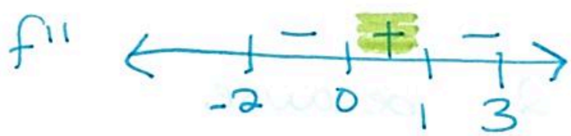
→ f' is DNE or 0

a) Find the x- values for any relative extrema. Identify if they are maximums or minimums and justify your conclusions.



Rel. max @ x=1 f' changes from + to −

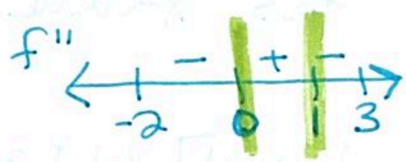
b) Where is $f(x)$ concave up? Justify your answer.



[0,1] $f'' \geq 0$ on this interval

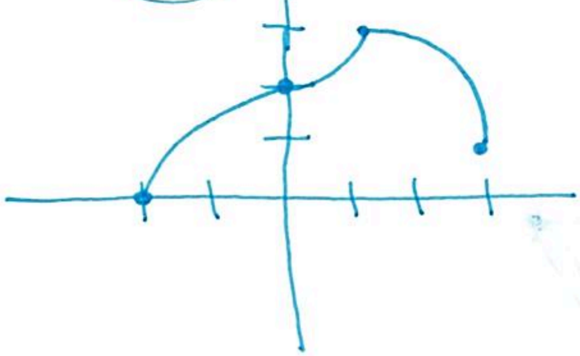
c) Find any points of inflection. Justify your answer.

POI at (0,2) and (1,3)

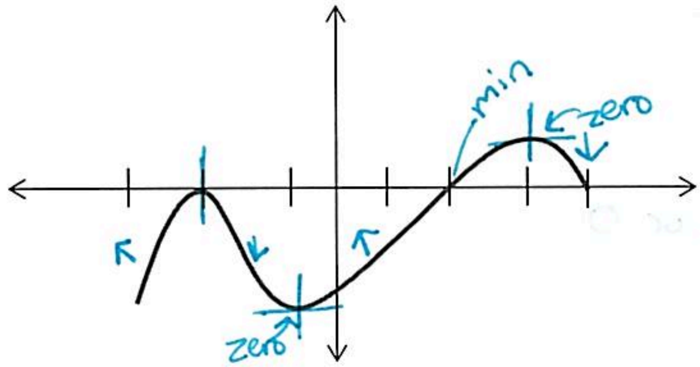


f'' changes signs at these points

d) Sketch a graph of $f(x)$ on [-2,3] that satisfies the given information.



2. A function $f(x)$ is continuous on [-3,4] and the graph of $f'(x)$ is given below.
$f(-3) = 2,\ f(-1) = 0,\ and\ f(4) = 0$



a) What are the critical numbers for $f(x)$? Justify your conclusion.

X= -2, 2, 4

f '(x)=0 at these x-values

b) Where does $f(x)$ have relative extrema? Is each extrema a relative maximum or a relative minimum? Justify your conclusion.

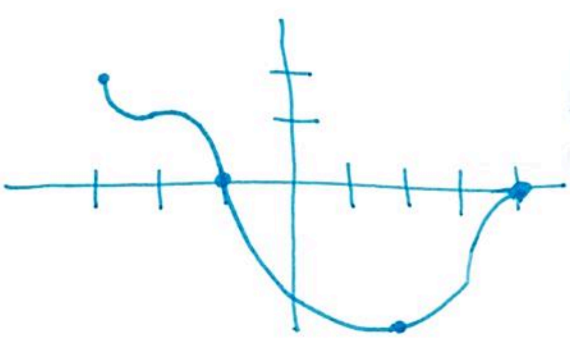
Rel. min for f(x) @ X=2 because f ' is changing from − to + at this x-value.

c) On what interval(s) is $f(x)$ concave down? Justify your conclusion.

[-2,-1] U [3,4]

f '(x) is decreasing so f(x) is concave down

d) Sketch a graph of $f(x)$ on [-3,4] that satisfies the given information.

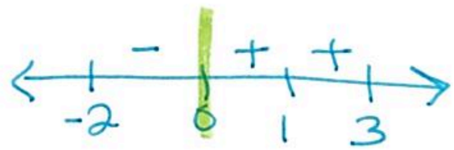


3. A function $f(x)$ is continuous on on [-2,3] and has the properties for $f'(x)$ and $f''(x)$ given below.

x	-2	-2<x<0	0	0<x<1	1	1<x<3	3
$f(x)$	0	Negative	-2	Negative	0	Positive	3
$f'(x)$	DNE	Negative	DNE	Positive	0	Positive	DNE
$f''(x)$	DNE	Negative	DNE	Negative	0	Positive	DNE

a) Find the x- values for any relative extrema. Identify if they are maximums or minimums and justify your conclusions.

f'



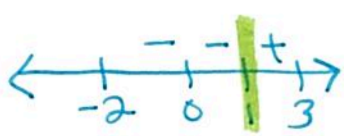
Rel. min @ x=0 f' changes from − to +

b) Where is $f(x)$ increasing? Justify your answer.

f(x) is increasing on [0, 3] because f'(x) ≥ 0 on this interval

c) Find any points of inflection. Justify your answer.

f''



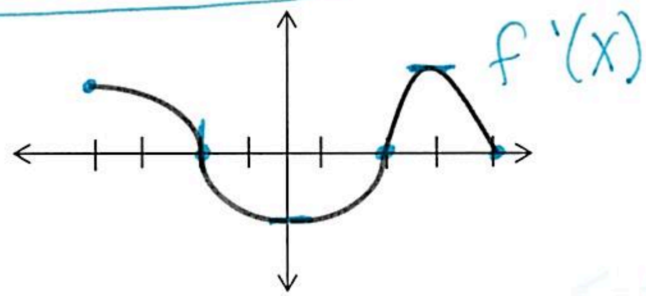
POI @ (1, 0) f'' changes signs at this poin

d) Sketch a graph of $f(x)$ on [-2,3] that satisfies the given information.

4. A function $f(x)$ is continuous on [-4,4] and the graph of $f'(x)$ is given below.

$f'(x)$ has a vertical tangent at x=-2 and horizontal tangents at x=0 and x=3.

$x = -4, 0, and\ 4\ are\ all\ roots\ of\ f(x).$



a) Where is $f(x)$ increasing? Justify your conclusion.

[-4, -2] ∪ [2, 4]

$f'(x) \geq 0$ on these intervals

b) Where does $f(x)$ have relative extrema? Is each extrema a relative maximum or a relative minimum? Justify your conclusion.

Rel. max @ x = -2 f ' changes from + to −

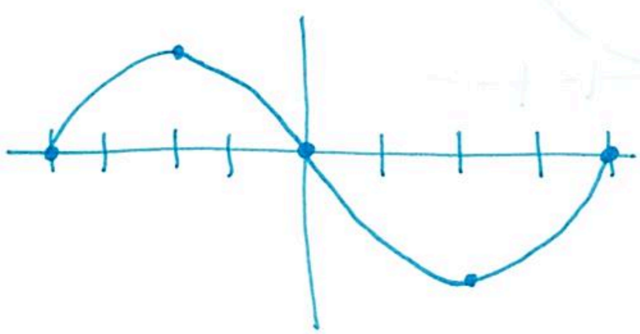
Rel. min @ x = 2 f ' changes from − to +

c) Where is $f(x)$ concave down? Justify your conclusion.

[-4, 0] ∪ [3, 4]

f ' is decreasing on these intervals

d) Sketch a graph of $f(x)$ on [-4,4] that satisfies the given information assuming that $f(0) = 0$.



Multiple Choice Practice –

*The answers are highlighted, you must justify the correct answer with proper AP justification.

1. If $f(x) = \sin\left(\frac{x}{2}\right)$, then there exists a number c in the interval $\frac{\pi}{2} < x < \frac{3\pi}{2}$ that satisfies the conclusion of the Mean Value Theorem. Which of the following could be c?

A) $\frac{2\pi}{3}$ B) $\frac{3\pi}{4}$ C) $\frac{5\pi}{6}$

D) π E) $\frac{3\pi}{2}$

AROC: $\dfrac{f\left(\frac{3\pi}{2}\right) - f\left(\frac{\pi}{2}\right)}{\frac{3\pi}{2} - \frac{\pi}{2}} = \dfrac{\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}}{\pi} = \frac{0}{\pi} = 0$

IROC: $f'(x) = \frac{1}{2}\cos\frac{x}{2}$

$\frac{1}{2}\cos\frac{x}{2} = 0$

$\cos\frac{x}{2} = 0$

$\frac{x}{2} = \frac{\pi}{2}, \frac{3\pi}{2}$

$\boxed{x = \pi}, 3\pi$

2. At what value of x does the graph of $y = \frac{1}{x^2} - \frac{1}{x^3}$ have a point of inflection?

A) 0 B) 1 C) 2

D) 3 E) At no value of x

$y = x^{-2} - x^{-3}$

$y' = -2x^{-3} + 3x^{-4}$

$y'' = 6x^{-4} - 12x^{-5} = \frac{6}{x^4} - \frac{12}{x^5}$

$= \frac{6x - 12}{x^5} = 0$

$6x - 12 = 0$

$x = 2$

$x^5 = 0$

$x = 0$

$+ \quad 0 \quad - \quad 2 \quad +$

VA @ $x = 0$ so DNE

3. The derivative of f is $x^4(x-2)(x+3)$. At how many points will the graph of f have a relative maximum?

A) none B) one C) two

D) three E) four

$f' = 0$

$x = 0, 2, -3$

$+ \quad -3 \quad - \quad 0 \quad - \quad 2 \quad +$

max (at -3), min (at 2)

4. How many critical points does the function $f(x) = (x+2)^5(x-3)^4$ have?

A) one B) two C) three

D) five E) nine

$f'(x) = (x-3)^4\left(5(x+2)^4\right) + (x+2)^5(4(x-3)^3)$

$f'(x) = (x-3)^3(x+2)^4\left[5(x-3) + 4(x+2)\right]$

① ②

$[5x - 15 + 4x + 8]$

$[9x - 7]$

③

5. Let f be the function with derivative given by $f'(x) = x^2 - \frac{5}{x}$, on which of the following intervals is f increasing. $f' \geq 0$

$$\frac{x^3 - 5}{x} = 0$$

A. $(-\infty, \infty)$

B. $(-\infty, 0) \cup (\sqrt[3]{5}, \infty)$

C. $(\sqrt[3]{5}, \infty)$ only

D. $(0, \sqrt[3]{5})$

CV: $x = 0, \sqrt[3]{5}$

f': $+$ | $-$ | $+$

$0 \quad \sqrt[3]{5}$

6. Let f be the function defined by $f(x) = \begin{cases} x^3, & x \leq 0 \\ x, & x > 0 \end{cases}$. Which of the following statements about f is true?

A) f is an odd function

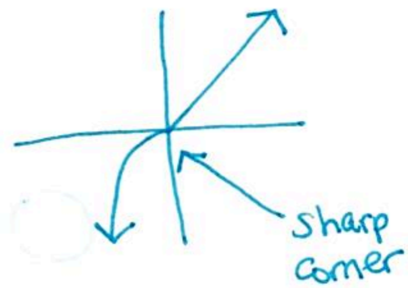
B) f is discontinuous at x=0

C) f has a relative maximum

D) $f'(0) = 0$

E) $f'(x) > 0$ for $x \neq 0$

$$f'(x) = \begin{cases} 3x^2, & x < 0 \\ 1, & x > 0 \end{cases}$$



$f'(x)$ does not exist @ $x=0$

$\lim_{x \to 0^-} f' = 0$

$\lim_{x \to 0^+} f' = 1$

Calculator questions

7.** If the derivative of f is given by $f'(x) = e^x - 3x^2$ at which of the following values of x does f have a realtive maximum value?

f' changes from $+$ to $-$

A. -0.46

B. 0.20

C. 0.91

D. 0.95

E. 3.73

$y_1 = e^x - 3x^2$

$y_2 = 0$

2nd – Trace – 5 – Enter 3 times

$x = .91$

8.** The function f is given by $f(x) = x^3 + 12x - 24$ is

$f'(x) = 3x^2 + 12$

$f'(x) \geq 0$ for all x

$\therefore$ $f(x)$ is always inc.

A) increasing for x<-2, decreasing for −2<x<2, increasing for x>2.

B) decreasing for x<0, increasing for x>0.

C) increasing for all x

D) decreasing for all x

E) decreasing for all x<-2, increasing for −2<x<2, decreasing for x>2.

9.** The function f has a first derivative given by $f'(x) = \dfrac{x}{x^2 - x - 1}$. What is the x coordinate of the inflection point of the graph of f ?

f'' changes signs @ POI

f'' is the slope of f'

the slope of f' never changes sign

A. -0.618

B. 1.618

C. 0

D. -4.866

E. The graph of f has no inflection point