U-Sub Day 2:

Remember

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

it follows that

$$\int \frac{1}{u}\,du = \ln|u| + C$$

Example:

$$\int \frac{2x}{x^2+1}\,dx$$

$u = x^2+1$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$$\int \frac{\cancel{2x}}{u} \cdot \frac{du}{\cancel{2x}} = \int \frac{1}{u} \cdot du = \ln|u| + C = \boxed{\ln(x^2+1) + C}$$

↳ * we don't need $|x^2+1|$ because $x^2+1 > 0$.

Example: $\int \frac{x^2}{3x^3+4}\,dx$

$u = 3x^3+4$

$\frac{du}{dx} = 9x^2$

$dx = \frac{du}{9x^2}$

$$\int \frac{\cancel{x^2}}{u} \cdot \frac{du}{9\cancel{x^2}} = \frac{1}{9}\int \frac{1}{u}\,du$$

$$= \frac{1}{9}\ln|u| + C$$

$$\boxed{= \frac{1}{9}\ln|3x^3+4| + C}$$

Example: $\int \tan x \, dx = \int \frac{\sin x}{\cos x} \, dx$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$dx = \frac{du}{-\sin x}$

$$= \int \frac{\cancel{\sin x}}{u} \cdot \frac{du}{-\cancel{\sin x}}$$

$$= -\int \frac{1}{u} \, du$$

$$= -\ln|u| + C$$

$$= \boxed{-\ln|\cos x| + C \quad \text{or} \quad \ln|\sec x| + C}$$

Example: $\int \cot x \, dx = \int \frac{\cos x}{\sin x} \, dx$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$dx = \frac{du}{\cos x}$

$$= \int \frac{\cancel{\cos x}}{u} \cdot \frac{du}{\cancel{\cos x}}$$

$$= \int \frac{1}{u} \, du$$

$$= \ln|u| + C = \boxed{\ln|\sin x| + C}$$

Example: $\int \sec x \, dx$

this one is tricky to make work. We have to multiply by a version of 1.

$$\int \sec x \cdot \frac{\sec x + \tan x}{\sec x + \tan x} \, dx = \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} \, dx$$

$u = \sec x + \tan x$

$\frac{du}{dx} = \sec x \tan x + \sec^2 x$

$dx = \frac{du}{\sec x \tan x + \sec^2 x}$

$$= \int \frac{\cancel{\sec^2 x + \sec x \tan x}}{u} \cdot \frac{du}{\cancel{\sec x \tan x + \sec^2 x}}$$

$$= \int \frac{du}{u}$$

$$= \ln |u| + C$$

$$\boxed{= \ln |\sec x + \tan x| + C}$$

Using the same process,

$$\boxed{\int \csc x \, dx = -\ln |\csc x + \cot x| + C}$$

Example: $\int \frac{\ln x}{x}\,dx$

whenever you see $\ln x$ in an integral, let $u = \ln x$.

$u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$dx = x \cdot du$

$$= \int \frac{u}{\cancel{x}} \cdot \cancel{x}\,du$$

$$= \int u\,du$$

$$= \frac{u^2}{2} + C$$

$$= \boxed{\frac{(\ln x)^2}{2} + C}$$

Double u-sub (AKA Advanced u-sub)

$$\int x\sqrt{x-3}\,dx = \int x \cdot u^{1/2}\,du$$

there is still an x!!!

since $u = x-3$
$x = u+3$

$u = x-3$

$\frac{du}{dx} = 1$

$dx = du$

$$= \int (u+3)\cdot u^{1/2}\,du$$

$$= \int u^{3/2}\,du + \int 3u^{1/2}\,du$$

$$= \frac{2}{5}u^{5/2} + \cancel{3}\cdot\frac{2}{\cancel{3}}u^{3/2} + C$$

$$= \boxed{\frac{2}{5}(x-3)^{5/2} + 2(x-3)^{3/2} + C}$$